

Problem Solving Defined and Used:
A Collection of Mathematical Problem-Solving Tasks

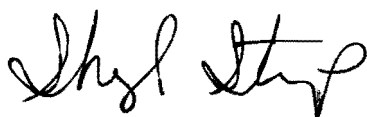
An Honors Thesis (HONRS 499)

by

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A handwritten signature in black ink, appearing to read "Sheryl Stump". The signature is fluid and cursive, with the first name "Sheryl" and last name "Stump" clearly distinguishable.

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Abstract

The following project includes a description of what problem solving is, how it is used in the mathematics classroom, and a collection of problem-solving tasks. Each problem is located in a section titled using one of the National Council of Teachers of Mathematics (NCTM) content standards: Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability. The major emphasis of the problems included here is on their ability to meet various Indiana state standards while still meeting specific requirements I set forth in building the collection, such as their real-world application, the amount of reading required, and the level of mathematics involved. The collection is intended to be a resource for teachers and student teachers to pull problems from and use in the classroom setting to promote problem solving.

Acknowledgements

I would like to thank Dr. Sheryl Stump for advising me throughout this project. Without her patient guidance, this project would never have gotten off of the ground.

I would also like to thank my family and friends for all of their support on this project and throughout my college experience.

“Learning to solve problems is the principle reason for studying mathematics” (Dolan & Williamson, 1983, ix). Although a mathematics setting is a great place to use problem solving and as Dolan’s quote implies a great opportunity to learn to solve problems, it is by no means the only setting where problem solving can or should be used. Rather, we use problem solving every day from situations that may arise in a classroom to those that involve deciding the best way to cut a cake into equal portions. The following project includes a description of what problem solving is, how it is used in the mathematics classroom, and a collection of problem-solving tasks. Each problem is located in a section titled using one of the National Council of Teachers of Mathematics (NCTM) content standards: Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability. The major emphasis of the problems included here is on their ability to meet various Indiana state standards while still meeting specific requirements I set forth in building the collection (outlined in THE COLLECTIONS section of this project). The collection is intended to be a resource for teachers and student teachers to pull problems from and use in the classroom setting to promote problem solving.

PROBLEM SOLVING

In building a collection of tasks that emphasize problem solving, it is especially important to define what is meant by the term and what it means to problem solve. Problem solving takes place in a variety of settings, however, the focus of this paper is on the problem solving and problem posing that takes place in the mathematics classroom.

The term “problem solving” is extremely hard to define. For instance, Webster’s dictionary broadly defines problem solving as “the thought processes involved in solving a problem” (Parker, 2006). Though this statement correctly addresses the key term involved in problem solving – process – it in no way defines what problem solving actually is. Yes, problem solving is a process, but there are many more details that need to be addressed. As NCTM states, “problem solving means engaging in a task for which the solution method is not known in advance” (NCTM, 2000, p. 52). Still another source states more specifically that problem solving is a “process by which an individual uses previously learned concepts, facts, and relationships, along with various reasoning skills and strategies, to answer a question or questions about a situation” (O’Daffer, Charles, Cooney, Dossey, & Schielack, 2005, p. 37). In other words, problem solving involves students using prerequisite knowledge to develop a method for arriving at a solution to a more complicated problem than they’ve previously encountered.

Similar complications also arise in defining exactly what a problem is. In order to problem solve, one must decide what exactly it means to be a “problem.” Before doing so, we must realize that “what is a problem to one person may not be a problem for another” (O’Daffer et al., 2005, p. 36). It is therefore necessary to define the term problem on a more general scale in hopes that several students in a class may experience the same feelings when faced with a problem task. The book *Mathematics for Elementary School Teachers* identifies a problem as “a situation for which the following conditions exist: (a) it involves a question that represents a challenge for the individual, (b) the question cannot be answered immediately by some routine procedures known to the individual, and (c) the individual [must] accept the challenge” (O’Daffer et al., 2005,

p. 36). It is obvious that not every problem could challenge each student in a classroom appropriately when ability levels and prerequisite knowledge are taken into account. The collection of problems here represents a compilation of real-world problems that involve complicated situations that a majority of students in the designated grade levels or courses might find problematic. Some problems might use routine procedures, however it is intended that these problems be used in a manner specific to the classroom that the problem solving is taking place in. For instance, a problem included in the algebra section focuses on the use of algebraic simplification. The problem may not present a challenge to a student in a second year algebra course, but it would certainly present an appropriate challenge to a student in late middle school or even perhaps in the first year of algebra.

Having identified what a problem is and what it means to problem solve aids us in determining the types of problems we should pose in order to promote problem solving in the mathematics classroom. It is necessary to note that posing a problem should take into account the prior knowledge of students and the level of difficulty they should be faced with given a specific task. It is also important to realize that problem solving is not a process that is automatically learned; rather students must be taught about problem solving and how to use it, only then may teachers use mathematical problems to aid in problem solving and mathematics comprehension.

Problem Solving in the Mathematics Classroom

There is a problem solving continuum that must be identified when discussing the use of problem solving. Patricia Baltzley, the Supervisor of Secondary Mathematics

Teachers for the Baltimore County Public Schools, states that problem solving is not an “isolated process but a process that starts with teaching students ABOUT problem solving, then preparing them FOR problem solving, and, finally, teaching them VIA problem solving” (Baltzley, 2005, para. 1). It is important that students learn about the problem-solving process and begin to understand the steps they can take to find a solution to a problem situation. The ultimate goal for the mathematics classroom is to use problematic situations to introduce students to new concepts or even to challenge students to use their existing knowledge in a different way.

In order to use problem solving in the mathematics classroom there are several steps mathematics teachers should take so as to provide students with the necessary skills to succeed in problem solving. It is important that students first build a bank of knowledge about how to solve problems. In other words, students must learn “how to approach something that they do not know the answer to in advance” (Baltzley, 2005). A problem I’ve run across several times in the mathematics classroom is the unwillingness for students to attempt something they see as challenging. Preparing students for problem solving by teaching them about the processes involved would aid students in developing a resistance to shutting down. I’ve seen many students, after being faced with a simple problem-of-the-day, merely shake their head and say “No, I won’t do it. I don’t know how.” It is this situation that represents the necessity to prepare students for applying what they already know to arrive at an appropriate solution to a problem.

Teaching students about problem solving involves methods that Polya described nearly 50 years ago. George Polya, “the father of problem solving,” described a simple list of steps one could take to begin to process the information in a problem situation

(Polya, 1945, p. xvi). First, students must understand the problem. Polya urged students to first be sure they understood the problem being posed by asking a series of questions about the problem: “What is the unknown? What are the data?” (Polya, 1945, p. xvi). Students must read and reread the problem, so that they are sure they understand what the problem is and what it is asking them to find. Secondly, Polya explains that students should devise a plan in which the steps for solving the problem should be outlined. Problem solvers may answer simple questions Polya puts forth at this stage like “Have you seen [this problem] before? Have you seen the same problem in a slightly different manner? Do you know a related problem? Could you restate the problem?” (Polya, 1945, p. xvi). It is important that students recognize what knowledge they might already have that would help them to solve the problem. Polya then requests that students carry out their plan. Lastly, students should look back and “examine the solution obtained” for reasonableness (Polya, 1945, p. xvi). Teaching students about problem solving prepares them for situations where a problem-solving approach might be necessary. That is merely the beginning; students must still learn the possible strategies to use when solving a problem.

A necessary component in the problem-solving process is development of a collection of strategies to use when a problem-solving situation arises. This step in the problem-solving continuum is what Baltzley calls teaching students “how to prepare FOR problem solving” (Baltzley, 2005, para. 1). To prepare students for problem solving they must have a variety of problem-solving strategies in which they can pull from and apply to a given problem situation. There are several problem-solving strategies including making a model, acting out the problem, choosing an operation, writing an equation,

drawing a diagram, guess-check-revise, simplifying the problem, making a list, looking for a pattern, making a table, using a specific case, working backward, and using reasoning (O'Daffer et al., 2005, p. 38). Students should be able to read a problem and identify the appropriate problem-solving strategy to apply. This may be one of the most difficult parts of problem solving. As students begin to dissect the problem in the understanding stage of problem solving, they may find it difficult to decide what to do next. As Polya's steps outlined, this is where the planning portion of problem solving comes into play. Students must be able to plan a strategy for finding a solution. They must identify what skills they will need and what strategies are most appropriate by utilizing information they already know. After building this repertoire of strategies, students have the opportunity to choose the appropriate methods to arrive at solutions to the types of problems they are faced with.

In the end, the dilemma we face as teachers is "Do [we] teach [problem solving] strategies directly or do [we] integrate them into the mathematics instruction?" (Baltzley, 2006, para. 3). The collection of problems presented here may be used in a variety of manners. These problems use an array of the problem-solving strategies discussed earlier; it is not the intention of the problems to focus on the specific method students use to obtain a solution, rather that the problems help students to become better problem solvers by introducing them to situations they may not immediately know how to find a solution to and are therefore forced to apply the steps of problem solving.

Ultimately, "the most important part of teaching problem solving is teaching VIA problem solving" (Baltzley, 2006, para. 1). Baltzley states that this "means that we use real world problems as the vehicle for learning mathematics" (Baltzley, 2005, para. 4).

This is exactly what this collection of problems is intended to be used for. Teachers may use the problems to aid students in becoming better problem solvers while also aiding in the enhancement of the mathematical skills involved in solving each problem.

Problem solving is an integral part of the mathematics classroom. As NCTM states, “By learning problem solving in mathematics, students . . . acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom” (NCTM, 2000, p. 52).

The Collection

Because the mathematics classroom is one of the best settings to allow students the opportunity to problem solve, the *Indiana Academic Standards for Mathematics* includes sections that specifically address students’ abilities to problem solve. Though the classroom is an almost ideal place for students to learn to become problem solvers, it is in no way the only environment where students will face problems. We are participants in an extremely complicated world where problem solving happens every day. It is this real world problem solving that drove me to seek a collection of problems that could be used in the mathematics classroom while also drawing on students abilities to problem solve outside of a mathematical context. Many of the tasks included in this collection involve real world questions that rely on mathematics to arrive at a solution.

The criteria for inclusion in this collection were simple. First and foremost, I looked for math problems that I found interesting. I strived to find problems that students might find interesting and would help them to see how much mathematics is used every day. I also took into account the amount of reading involved in the description of the

problem. If there was more reading involved than what I could handle, I was assured that students would also find the amount of reading cumbersome. I also took into account the type of problem that students were being asked to do. Because I wanted to emphasize real-world problems, I strived to find problems that included interesting real-world story situations. Of course, some problems did not meet the real-world aspect but I felt that they were equally intriguing and so included them also. After compiling a large collection of problems, I weeded out problems that were similar. The end result is still a large collection of tasks that can be used in the mathematics classroom to promote problem solving. One must realize that this collection is in no way a complete resource of problems meant to meet every standard put forth by NCTM at every math level. Instead, I searched for problems that fulfilled the criteria I was using and that met main components of the standards.

Because the problems are intended to be used to promote problem solving, students are required to explain their thoughts and ideas while working through the problems put forth. For each of the following problems, students are expected to provide the following information:

I. Problem Statement

- State the problem in your own words. Write the information clearly enough so that someone picking up your paper could easily understand what you were asked to do.

II. Plan

- Tell what you did to prepare to solve the problem. How did the problem seem to you when you first read it? Consider what you are asked to find, what you knew

before hand, what you need to know, and what strategies you can use. Is this problem like any others you've done?

III. Work

- Explain in detail what you did to solve the problem. Use charts and graphs where appropriate. Tell what worked, what didn't work, and what you did when you got stuck. Did you get help from anyone? What kind of help?

IV. Answer

- State your answer(s) to the problem. Does the answer make sense? Could there be other correct answers? Compare your find answer with your original guess. What did you learn from this problem that could help you to solve other problems? (Tsuruda, 1994, p. 41).

The rubrics that precede each collection of problems are intended to be used to evaluate students on their abilities to complete the required components of each problem. That is, students are expected to outline their process using the information above while also demonstrating proficiency in the mathematical concepts involved in the problems.

The following is the collection of problems in order of mathematical content: Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability.

NUMBER AND OPERATIONS PROBLEMS

NUMBER AND OPERATIONS RUBRIC

CATEGORY	1	2	3	4
Mathematical Concepts	Explanation shows very limited understanding of the concepts involved OR is not written. Student does not meet number and operations expectations or standards.	Explanation shows some understanding of the concepts needed to solve the problem. Student may partially meet standards and expectations addressed in the problem.	Explanation shows substantial understanding of the concepts used to solve the problem. Student demonstrates adequate fulfillment of expectations and meets standards.	Explanation shows complete understanding of the concepts used to solve the problem. Student demonstrates fulfillment of expectations and meets standards.
Mathematical Errors	More than 75% of the steps and solutions have mathematical errors.	Most (75-84%) of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solutions have no mathematical errors.	90-100% of the steps and solutions have no errors.
Explanation	Explanation is difficult to understand and is missing several components OR was not included.	Explanation is a little difficult to understand, but includes critical components.	Explanation is clear.	Explanation is detailed and clear.
Mathematical Terminology and Notation	There is little use, or a lot of inappropriate use, of terminology and notation.	Correct terminology and notation is used, but it is sometimes not easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminology and notation are always used, making it easy to understand what was done.
Neatness and Organization	The work appears sloppy and unorganized. It is hard to determine what information goes together.	The work is presented in an organized fashion but may be hard to read at times.	The work is presented in a neat and organized fashion that is usually easy to read.	The work is presented in a neat, clear, organized fashion that is always easy to read.

Number and Operations 6-8:

- Work flexibly with fractions, decimals, and percents to solve problems
- Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers
- Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods

Indiana State Standards:

- 6.2.3 Multiply and divide decimals.
- 6.2.7 Understand proportions and use them to solve problems.
- 6.2.8 Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips.
- 7.2.1 Solve addition, subtraction, multiplication, and division problems that use integers, fractions, decimals, and combinations of the four operations.
- 7.2.3 Solve problems that involve discounts, markups, and commissions.
- 8.2.1 Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) in multi-step problems.

The Problem:

*The case cost of Gallo Pepperoni at Albertson's grocery store is \$24 per 12 (8 oz.) packages. The warehouse markup will be 7%. The store markup will be 38%. What is the retail price at of a package of pepperoni?**

Mathematical Content:

This task requires students to use their knowledge of proportions to determine the cost of an individual package of pepperoni. Upon finding the cost of an individual package, students must determine given percentages of this cost. Students must multiply decimals to determine markup prices and eventually add several decimals together.

Solution:

$$\text{Package cost} = \frac{\$24.00}{12 \text{ packages}} = \$2.00 / \text{package}$$

$$\text{Warehouse markup} = \$2.00 \cdot .07 = \$0.14$$

$$\text{Albertson's markup} = \$2.14 \cdot .38 = \$0.81$$

$$\text{Albertson's retail price} = \$2.00 + \$0.14 + \$0.81 = \$2.95 / \text{package}$$

* Source: Micron Technology, Inc. (2005). Math in the Workplace: Number and Operations Problems. *Math in the Workplace*. Retrieved August 15, 2005, from <http://www.micron.com/k12/math/numop/foodbroker.html>

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- 7.2.3 Solve problems that involve discounts, markups, and commissions.
- 8.2.1 Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) in multi-step problems.

The Problem:

All employees must have Federal Social Security (FICA) and Medicare taxes taken from their gross wages. Figure the FICA and Medicare taxes deducted from the following employees' gross wages and for the total gross wages. Lastly, determine the take home pay of each employee.

<i>Employee A</i>	<i>\$540.00</i>
<i>Employee B</i>	<i>\$870.00</i>
<i>Employee C</i>	<i>\$250.00</i>
<i>Employee D</i>	<i>\$125.00</i>
<i>Employee E</i>	<i>\$1600.00</i>

Gross wages refer to the total amount earned before any deductions.

FICA is 6.2% of the gross wages.

Medicare is 1.45% of the gross wages.

*Take home pay refers to the amount an employee actually receives after FICA and Medicare taxes have been deducted.**

* Source: Micron Technology, Inc. (2005). Math in the Workplace: Number and Operations Problems. *Math in the Workplace*. Retrieved August 15, 2005, from <http://www.micron.com/k12/math/numop/bookkeeper1.html>

Mathematical Content:

This task requires students to work with decimals in a number of ways. Students must use multiplication of decimals to find the deductions taken from total gross wages and each employee. Students must also subtract decimal numbers to determine the take home pay of each employee.

Solution:

	540.00
	870.00
	250.00
Total Gross Wages	125.00
	+1600.00
	<hr/>
	3385.00

$$\text{FICA} = .062 \cdot \$3385.00 = \$209.87$$

$$\text{Medicare} = .0145 \cdot \$3385.00 = \$49.08$$

Deductions from each employee for FICA and Medicare

<i>A</i>	$.062 \cdot 540.00 = \$33.58$	$.0145 \cdot 540.00 = \$7.83$
<i>B</i>	$.062 \cdot 870.00 = \$53.94$	$.0145 \cdot 870.00 = \$12.62$
<i>C</i>	$.062 \cdot 250.00 = \$15.50$	$.0142 \cdot 250.00 = \$3.63$
<i>D</i>	$.062 \cdot 125.00 = \$7.75$	$.0142 \cdot 125.00 = \$1.81$
<i>E</i>	$.062 \cdot 1600.00 = \$99.20$	$.0142 \cdot 1600.00 = \$23.20$

Take home pay

<i>A</i>	$\$540.00 - \$33.58 - \$7.83 = \498.59
<i>B</i>	$\$870.00 - \$53.94 - \$12.62 = \803.44
<i>C</i>	$\$250.00 - \$15.50 - \$3.63 = \230.87
<i>D</i>	$\$125.00 - \$7.75 - \$1.81 = \115.44
<i>E</i>	$\$1600.00 - \$99.20 - \$23.20 = \1477.60

Number and Operations 6-8:

- Work flexibly with fractions, decimals, and percents to solve problems
- Understand and use ratios and proportions to represent quantitative relationships
- Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods

Indiana State Standards:

- 6.2.5 Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.
- 6.2.6 Interpret and use ratios to show the relative sizes of two quantities. Use the notations: a/b , a to b , $a:b$.
- 6.2.7 Understand proportions and use them to solve problems.
- 7.2.1 Solve addition, subtraction, multiplication, and division problems that use integers, fractions, decimals, and combinations of the four operations.

The Problem:

Do you always get 6 hours of recording on a 6 hour tape?

*Suppose the setting SP (standard play) on a VCR allows 2 hours of recording with an ordinary 120 minute tape. Changing the setting to EP (extended play) allows 6 hours of recording. After taping a 30 minute show on SP, the VCR is reset to EP. How many more 30 minute shows can be recorded on this tape?**

Mathematical Content:

This task involves various types of proportional reasoning. Students must use proportions to convert the number of minutes used to hours used or the number of hours available to minutes available. Once students have determined the amount of time used in the SP setting, they can then determine the amount of time left and decide how many 30 minute shows can be taped in that time.

Solution:

You can consider this problem in terms of minutes or in units of half hour shows. Using only the SP setting, the tape can record 120 minutes of shows. If one 30 minute show has already been recorded, $\frac{1}{4}$ of the tape has been used. With $\frac{1}{4}$ of the tape filled, 90 minutes of EP has been used, so that there are $360 - 90$ or 270 minutes left. At 30 minutes per show, nine additional shows can be taped.

If you are using half-hour units, there are four half-hour shows on SP in two hours. One half-hour show used $\frac{1}{4}$ of the tape. Three-fourths of the tape is left. With EP you can

* Source: National Council of Teachers of Mathematics. (2004). Math Challenge #47, VCR. *Figure This!* Retrieved August 8, 2005, from <http://www.figurethis.org/challenges/c47/challenge.htm>

tape 12 half-hour shows. Upon subtracting the $\frac{1}{4}$ of the tape that has already been used, you have enough tape for $\frac{3}{4} \cdot 12$ or 9 shows remaining.

Additional Solution:

Yet another way to consider this problem is to use proportions as follows:

$$\frac{30 \text{ min}}{120 \text{ min on SP}} = \frac{? \text{ min}}{360 \text{ min on EP}} \quad \text{Because } 3 \cdot 120 = 360 \quad \text{and} \quad 3 \cdot 30 = 90, \text{ there are 90}$$

minutes used on the EP setting, so there are $360 - 90$, or 270 minutes available, which allows for 9 thirty-minute shows on EP.

Similarly $30 \cdot 360 = 120 \cdot (\text{number of min})$ so the number of minutes used is 90.

Number and Operations 6-8:

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Indiana State Standards:

- 6.2.3 Multiply and divide decimals.
- 6.2.8 Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips.
- 7.2.1 Solve addition, subtraction, multiplication, and division problems that use integers, fractions, decimals, and combinations of the four operations.
- 7.2.3 Solve problems that involve discounts, markups, and commissions.
- 7.2.5 Use mental arithmetic to compute with simple fractions, decimals, and powers.
- 8.2.4 Use mental arithmetic to compute with common fractions, decimals, powers, and percents.

The Problem:

So how much does it cost?

*Is a discount of 30% off the original price, followed by a discount of 50% of the sale price, the same as a discount of 80% from the original price?**

Mathematical Content:

In this task students must calculate the price of merchandise with various percentages of discounts. Students must develop a plan for analyzing the amount of money saved at the various levels of discount. After finding the value saved at each discount level, students should easily be able to decide whether or not the discounts are the same.

Solution:

If an item originally costs \$100, the tables below show the different final costs. They are not the same.

Original Price	30% Off	Cost on Sale	50% Off Sale Price	Final Cost
\$100	$30\% \cdot \$100 = \30	$\$100 - \$30 = \$70$	$50\% \cdot \$70 = \35	$\$70 - \$35 = \$35$

Original Price	80% Off	Final Cost
\$100	$80\% \cdot \$100 = \80	$\$100 - \$80 = \$20$

For the item on sale at 30% off, you would need to pay 70% of the price. So an additional Discount of 50% off the sale price would bring the price to 35% (that is,

* Source: National Council of Teachers of Mathematics. (2004). Math Challenge #17, Perplexing Percentages. *Figure This!* Retrieved August 8, 2005, from <http://www.figurethis.org/challenges/c17/challenge.htm>

$50\% \cdot 70\%$) off the original price. Thus, a \$100 item would cost \$35 after both discounts. An 80% off sale means that you pay $100\% - 80\%$, or 20% of the original cost of the item. Thus, an item that originally cost \$100 on sale at 80% off costs $20\% \cdot \$100$ or \$20. The costs are not the same.

You can also generalize this problem. If P is the original price of an item, with the two discounts, one of the 30% followed by another of 50%, you would pay $0.50 \cdot (0.70 \cdot P)$ or $0.35P$, which is not the same as $0.20P$.

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Indiana State Standards:

- 6.2.3 Multiply and divide decimals.
- 6.2.8 Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips.
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- 7.2.3 Solve problems that involve discounts, markups, and commissions.
- 7.2.5 Use mental arithmetic to compute with simple fractions, decimals, and powers.
- 8.2.1 Add, subtract, multiply, and divide rational numbers (integers*, fractions, and terminating decimals) in multi-step problems.
- 8.2.4 Use mental arithmetic to compute with common fractions, decimals, powers, and percents.

The Problem:

Do movies make money?

*The theater box office receipts for the movie "Ratio in Magicville" for the past four weeks were \$15,000, \$12,000, and \$10,000, respectively. The theater owner pays the movie distributor the following percentages of the box office receipts: 70% for each of the first two weeks, 60% for the third week, and 50% for the fourth week. Other operating expenses are \$4500 per week. Did the theater make or lose money?**

Mathematical Content:

In this task students are asked to determine whether a company has experienced a profit or a loss. Students must keep track of the percentage of profit that must be turned over to the movie distributor in a given week. They must eventually determine the amount of money the movie theatre has as a profit or loss for each week. After finding the amount of profit or loss per week, students can determine the amount of total profit or loss for the theatre.

Solution:

For each week, subtract the distributor's share and other expenses from the theater box-office receipts. The result is the weekly profit (or loss). See the table on the following page. After four weeks, the theater has a loss of \$100.

* Source: National Council of Teachers of Mathematics. (2004). Math Challenge #77, Move Money. *Figure This!* Retrieved August 8, 2005, from <http://www.figurethis.org/challenges/c77/challenge.htm>

Week	Box-Office Receipts, R	Percentage p , to Distributor	Amount, $A=Rp$ to Distributor	Other Expenses, E	Profit or Loss $R - P = E$
1	\$15,000	70%	\$10,500	\$4500	\$0
2	\$12,000	70%	\$8400	\$4500	-\$900
3	\$12,000	60%	\$7200	\$4500	\$300
4	\$10,000	50%	\$5000	\$4500	\$500
				Total	-\$100

Number and Operations 6-8:

- Work flexibly with fractions, decimals, and percents to solve problems
- Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods

Indiana State Standards:

- 6.2.3 Multiply and divide decimals.
- 6.2.6 Interpret and use ratios to show the relative sizes of two quantities. Use the notations: a/b , a to b , $a:b$.
- 6.2.7 Understand proportions and use them to solve problems.
- 7.2.1 Solve addition, subtraction, multiplication, and division problems that use integers, fractions, decimals, and combinations of the four operations.
- 8.2.1 Add, subtract, multiply, and divide rational numbers (integers*, fractions, and terminating decimals) in multi-step problems.
- 8.2.4 Use mental arithmetic to compute with common fractions, decimals, powers, and percents.

The Problem:

*Pedro and Sarah have 70 movies. If they want to watch the movies one right after another, and if each movie is approximately 1.5 hours long, how many days would it take Pedro and Sarah to watch all of the movies?**

Mathematical Content:

In this task students must multiply decimals and use unit conversions in order determine the amount of time it will take Pedro and Sarah to watch all 70 movies. First students must multiply decimals in order to find the total number of hours it will take Pedro and Sarah to watch all of the movies. After determining the total time they need to spend watching movies, students must determine the number of days it will take the two characters to watch all of the movies.

Solution:

4.375 days

$$70 \text{ movies} \cdot 1.5 \text{ hours} = 105 \text{ hours}$$

$$105 \text{ hours} \cdot \frac{1 \text{ day}}{24 \text{ hours}} = 4.375 \text{ days}$$

$$4 \text{ days} + \left(.375 \text{ days} \cdot \frac{1 \text{ day}}{24 \text{ hours}} \right) = 4 \text{ days and } 9 \text{ hours}$$

* Source: McGivney-Burelle, J. & White, J. (2005). February Calendar. *Mathematics Teacher*, 98 (6), 408.

Number and Operations 6-8:

- Use factors, multiples, prime factorization, and relatively prime numbers to solve problems

Indiana State Standards:

- 6.1.7 Find the least common multiple and the greatest common factor of whole numbers.
- 6.2.2 Multiply and divide positive and negative integers.
- 7.1.4 Understand and compute whole number powers of whole numbers.
- 7.2.1 Solve addition, subtraction, multiplication, and division problems that use integers, fractions, decimals, and combinations of the four operations.
- 7.2.5 Use mental arithmetic to compute with simple fractions, decimals, and powers.
- 8.2.1 Add, subtract, multiply, and divide rational numbers (integers*, fractions, and terminating decimals) in multi-step problems.

The Problem:

*A school has a hall with 1000 lockers, all of which are closed. One thousand students start down the hall. The first student opens every locker. The second student closes all the lockers that are multiples of 2. The third student changes (closes an open locker or opens a closed one) all multiples of 3. The fourth student changes all multiples of 4, and so on. After all students have finished with the lockers, how many lockers are closed?**

Mathematical Content:

This task requires students to think critically about how to solve a problem that may at first seem very difficult. This particular problem can be broken down into easier steps and scenarios. Students should think about the problem in terms of 10 lockers and what happens to them when the lockers are opened and closed as described. Upon examining the outcome of situation on 10 lockers, students should discover that the lockers remaining open are those that have an odd number of factors. Once students learn the number of lockers opened (the perfect squares between 1 and 31) the problem is reduced to simple subtraction.

Solution:

969 lockers will be closed. Many strategies can be used to solve this problem. One such strategy uses a similar but smaller problem. Let's try the problem using the first ten lockers, where O means open and C designates closed. Before the students begin, the lockers are all closed, as shown below.

Locker	1	2	3	4	5	6	7	8	9	10
O/C	C	C	C	C	C	C	C	C	C	C

The position of all lockers after the first student passes all ten lockers:

Locker	1	2	3	4	5	6	7	8	9	10
--------	---	---	---	---	---	---	---	---	---	----

* Source: Rock, D. (2005). Menu of Problems. *Mathematics Teaching in the Middle School*, 11 (2), 24.

O/C	O	O	O	O	O	O	O	O	O	O
-----	---	---	---	---	---	---	---	---	---	---

The position of all the lockers after the second student passes all ten lockers and changes all multiples of two:

Locker	1	2	3	4	5	6	7	8	9	10
O/C	O	C	O	C	O	C	O	C	O	C

The position of all lockers after the third student passes all ten lockers and changes all multiples of three:

Locker	1	2	3	4	5	6	7	8	9	10
O/C	O	C	C	C	O	O	O	C	C	C

The position of all lockers after the 4th pass:

Locker	1	2	3	4	5	6	7	8	9	10
O/C	O	C	C	O	O	O	O	O	C	C

The position of all lockers after the 5th pass:

Locker	1	2	3	4	5	6	7	8	9	10
O/C	O	C	C	O	C	O	O	O	C	O

The position of all lockers after the 6th pass:

Locker	1	2	3	4	5	6	7	8	9	10
O/C	O	C	C	O	C	C	O	O	C	O

The position of all lockers after the 10th pass:

Locker	1	2	3	4	5	6	7	8	9	10
O/C	O	C	C	O	C	C	C	C	O	C

Notice that the open lockers are 1, 4, and 9, which are all perfect squares. This suggests that after 1000 students pass 1000 lockers, all perfect square number lockers less than or equal to 1000 will be open. Can we be sure? Yes. The fact is these numbers are the only ones for which a number is multiplied by itself means that they are the only ones touched an odd number of times, so they will be the only ones changed from “closed” to “open.”

Trial and error show that $32^2 = 1024$, whereas $31^2 = 961$. Therefore 31 lockers will be open, the squares of the numbers 1-31. The remaining 969 will be closed.

Number and Operations 6-8:

- Use factors, multiples, prime factorization, and relatively prime numbers to solve problems

Indiana State Standards:

- 6.2.2 Multiply and divide positive and negative integers.
- 7.1.4 Understand and compute whole number powers of whole numbers.
- 7.2.1 Solve addition, subtraction, multiplication, and division problems that use integers, fractions, decimals, and combinations of the four operations.
- 8.2.4 Use mental arithmetic to compute with common fractions, decimals, powers, and percents.

The Problem:

*The counting numbers greater than 1 are listed in five columns, as shown in the array below. Following the pattern, determine the column in which the number 1000 will appear. Justify your answer with a logical explanation.**

<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>	<u>E</u>
	2	3	4	5
9	8	7	6	
	10	11	12	13
17	16	15	14	
	18	19	20	21
25	24	23	22	

.

Mathematical Content:

This problem requires students to investigate patterns. This task could be completed by continuing the table until students arrive at the number 1000. That, however, would be a time consuming task. Instead, students might try looking for multiples of numbers in each column. For instance, the pattern for multiples of 4 includes columns D, B, D, B, D, B, ... This problem also requires students to determine the factors of 1000.

Solution:

1000 will appear in column B. There are many possible explanations. You could continue the pattern to locate 1000. A quicker method is to notice that all multiples of 8 are in column B. Since 1000 is a multiple of 8, it will also be in column B. You can test this theory by continuing the pattern to locate 32 and 40.

* Source: Rock, D. (2005). Menu of Problems. *Mathematics Teaching in the Middle School*, 11 (1), 24.

Number and Operations 6-8:

- Work flexibly with fractions, decimals, and percents to solve problems
- Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods

Indiana State Standards:

- 6.2.3 Multiply and divide decimals.
- 6.2.8 Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips.
- 7.2.1 Solve addition, subtraction, multiplication, and division problems that use integers, fractions, decimals, and combinations of the four operations.
- 7.2.2 Calculate the percentage increase and decrease of a quantity.
- 7.2.3 Solve problems that involve discounts, markups, and commissions.
- 8.2.1 Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) in multi-step problems.

The Problem:

*You are at a movie rental store that is selling two of your favorite DVDs. One is brand new, whereas the other is a previously viewed DVD. The new DVD costs \$22 and the previously viewed DVD is \$12. Today, all new DVDs are 20% off and all previously viewed DVDs are 40% off. Excluding tax, what will be the total price of the two DVDs after the discount and what will be the percent discount on the total purchase?**

Mathematical Content:

This task requires students to determine the costs of various items after a certain amount of discount has been taken. Using multiplication of decimals, students should arrive at the cost of the DVDs after their discounts. Students must then determine the percentage discount on their total purchase.

Solution:

\$24.80 with a 27 percent total discount. The cost of the new DVD is \$22.00 and is discounted 20 percent. The discount will be \$4.40. The discounted price will be $\$22.00 - \$4.40 = \$17.60$. The cost of the previously viewed DVD is \$12.00, with a 40 percent discount. The discount will be \$4.80. The discounted price will be $\$12.00 - \$4.80 = \$7.20$. The total price after discounts will be $\$17.60 + \$7.20 = \$24.80$. The full retail price without discounts is $\$22.00 + \$12.00 = \$34.00$. The total savings after the discounts is $\$34.00 - \$24.80 = \$9.20$. To determine the percent discount of the total purchase, divide the discount by the original price, $\frac{\$9.20}{\$34.00} \approx .27$ or 27%.

* Source: Rock, D. (2005). Menu of Problems. *Mathematics Teaching in the Middle School*, 11 (1), 24.

Number and Operations 6-8:

- Work flexibly with fractions, decimals, and percents to solve problems
- Understand and use ratios and proportions to represent quantitative relationships

Indiana State Standards:

- 6.2.1 Add and subtract positive and negative integers.
- 6.2.2 Multiply and divide positive and negative integers.
- 6.2.4 Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.
- 6.2.5 Interpret and use ratios to show the relative sizes of two quantities. Use the notations: a/b , a to b , $a:b$.
- 6.2.6 Understand proportions and use them to solve problems.
- 7.2.1 Solve addition, subtraction, multiplication, and division problems that use integers, fractions, decimals, and combinations of the four operations.
- 7.2.4 Use mental arithmetic to compute with simple fractions, decimals, and powers.
- 8.2.1 Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) in multi-step problems.
- 8.2.4 Use mental arithmetic to compute with common fractions, decimals, powers, and percents.

The Problem:

*How long will it take a mile-long train going 20 miles per hour to get completely through a 2 mile-long tunnel?**

Mathematical Content:

This problem requires students to think critically about how long it will take a train to pass through a tunnel that is longer than itself. Students may easily find the time it takes the front of the train to pass through the tunnel using unit conversions. Students must remember, however, that the end of the train will still be in the tunnel. Upon visualizing the actions of the train, students should easily be able to determine the amount of time needed in order for the entire train to pass through the tunnel.

Solution:

9 minutes.

If the train is going 20 mph, it will travel 1 mile in 3 minutes. $\frac{20 \text{ miles}}{1 \text{ hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} = \frac{1 \text{ mi}}{3 \text{ min}}$

It will take the engine 6 minutes to go through the tunnel (2 times the amount of time to travel 1 mile). However, it will take another 3 minutes for the caboose to go the additional 1 mile to clear the tunnel.

* Source: Rock, D. (2005). Menu of Problems. *Mathematics Teaching in the Middle School*, 11 (1), 24.

Number and Operations 9-12:

- Compare and contrast the properties of numbers and number systems, including the rational and real numbers, and understand complex numbers as solutions to quadratic equations that do not have real solutions
- Use number-theory arguments to justify relationships involving whole numbers

Indiana State Standards:

A1.6.1 Add and subtract polynomials.

A2.5.3 Factor polynomials completely and solve polynomial equations by factoring.

A2.5.5 Use polynomial equations to solve word problems.

The Problem:

*What is the greatest whole number that **MUST** be a factor of the sum of any four consecutive positive odd numbers?**

Mathematical Content:

In this task students must use their knowledge of odd and even numbers to develop an algebraic expression for each of the four consecutive positive odd numbers. Students must then use their algebraic expressions to obtain a single expression for the total value of the four consecutive positive odd numbers. Once students have determined this expression the problem is reduced to a problem involving factoring a polynomial.

Solution:

8. Let $2n+1$ be a positive odd number. Then $2n+3$, $2n+5$, and $2n+7$ represent the next three consecutive positive odd numbers. The sum of these four consecutive positive odd numbers is $8n+16$. Factoring an 8 from this expression gives $8(n+2)$, which implies that 8 is the greatest whole number that **MUST** be a factor of the sum of four consecutive positive odd numbers.

* Source: McGivney-Burelle, J. & White, J. (2005). November Calendar. *Mathematics Teacher*, 99 (4), 264.

Number and Operations 9-12:

- Compare and contrast the properties of numbers and number systems, including the rational and real numbers, and understand complex numbers as solutions to quadratic equations that do not have real solutions
- Use number-theory arguments to justify relationships involving whole numbers

Indiana State Standards:

A1.1.1 Compare real number expressions.

The Problem:

*Find the remainder when 2^{495} is divided by 11.**

Mathematical Content:

This problem requires students to determine the remainder of a large number when divided by 11. In order to find the solution to this problem, students should think of it in steps. Students must recall the properties of exponents in order to break down the exponent in the number 2^{495} into smaller, more manageable, parts. Once they have done so, students can easily determine the remainder for smaller powers of 2 when divided by 11.

Solution:

10. The table shows consecutive powers of 2 in the first column and the remainder when each power is divided by 11 in the second column. Therefore $2^{10} \equiv 1 \pmod{11}$ and $(2^{10})^{49} \equiv (1)^{49} \pmod{11}$, which implies that $2^{490} \equiv 1 \pmod{11}$. We can write 2^{495} as $2^{490}2^5 = 1 \times 2^5 \pmod{11}$ and we have $2^{495} \equiv 2^5 \pmod{11}$. From the table $2^5 \equiv 10 \pmod{11}$, so the remainder will be 10.

Power of 2	Remainder when Divided by 11
0	1
1	2
2	4
3	8
4	5
5	10
6	9
7	7
8	3
9	6
10	1

* Source: McGivney-Burelle, J. & White, J. (2005). November Calendar. *Mathematics Teacher*, 99 (4), 264.

ALGEBRA PROBLEMS

ALGEBRA RUBRIC

CATEGORY	1	2	3	4
Mathematical Concepts	Explanation shows very limited understanding of the algebraic concepts involved OR is not written. Student does not meet algebra expectations or standards.	Explanation shows some understanding of the algebraic concepts needed to solve the problem. Student may partially meet standards and expectations addressed in the problem.	Explanation shows substantial understanding of the algebraic concepts used to solve the problem. Student demonstrates adequate fulfillment of expectations and meets standards.	Explanation shows complete understanding of the algebra concepts used to solve the problem. Student demonstrates fulfillment of expectations and meets standards.
Mathematical Errors	More than 75% of the steps and solutions have mathematical errors.	Most (75-84%) of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solutions have no mathematical errors.	90-100% of the steps and solutions have no errors.
Explanation	Explanation is difficult to understand and is missing several components OR was not included.	Explanation is a little difficult to understand, but includes critical components.	Explanation is clear.	Explanation is detailed and clear.
Mathematical Terminology and Notation	There is little use, or a lot of inappropriate use, of algebraic terminology and notation.	Correct terminology and notation is used, but it is sometimes not easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminology and notation are always used, making it easy to understand what was done.
Neatness and Organization	The work appears sloppy and unorganized. It is hard to determine what information goes together.	The work is presented in an organized fashion but may be hard to read at times.	The work is presented in a neat and organized fashion that is usually easy to read.	The work is presented in a neat, clear, organized fashion that is always easy to read.

Algebra 6-8:

- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules
- Model and solve contextualized problems using various representations, such as graphs, tables, and equations

Indiana State Standards:

- 7.1.4 Understand and compute whole number powers of whole numbers.
- 7.3.9 Identify functions as linear or nonlinear and examine their characteristics in tables, graphs, and equations.

The Problem:

What's round, hard, and sold for \$3 million?

*Mark McGwire became baseball's home run king in 1998 with 70 home runs. His 70th home run ball sold for slightly over \$3 million in 1999. Babe Ruth, an earlier home-run king, hit 60 in 1927. His home-run ball was donated to the Hall of Fame. Suppose that Ruth's ball was valued at \$3000 in 1927 and, like many good investments, doubled its value every seven years. Would you rather have the value of Ruth's ball or McGwire's?**

Mathematical Content:

This task requires that students discover a pattern to determine the amount Babe Ruth's baseball is worth today. This is primarily a task involving patterns, but students must discover the pattern on their own. Students are given that the value of the ball doubles every year, but they must determine the amount of increase for following years.

Solution:

Suppose Ruth's ball had a value of \$3000 in 1927. If the price doubled in seven years, the ball would be worth \$6000 in 1934. In seven more years, its value would double again.

Year	Value
1927	3000
1934	$2 \cdot 3000 = 6000$
1941	$2 \cdot 2 \cdot 3000 = 2^2 \cdot 3000 = 12,000$
1948	$2 \cdot 2 \cdot 2 \cdot 3000 = 2^3 \cdot 3000 = 24,000$
1955	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3000 = 2^4 \cdot 3000 = 48,000$
	...
1997	$2^{10} \cdot 3000 = 3,072,000$

* Source: National Council of Teachers of Mathematics. (2004). Math Challenge #11, Play Ball. *Figure This!* Retrieved August 8, 2005, from <http://www.figurethis.org/challenges/c11/challenge.htm>

The year 1997 was 70 years after 1927 so there would be 10 sets of 7 years during that time. By 1997, Ruth's ball would have a value of \$3,072,000. Since it would have a greater value than McGwire's in 1997, it would have also a greater value in 2005.

Algebra 6-8:

- Represent, analyze, and generalize a variety of patterns, with tables, graphs, words, and when possible, symbolic rules

Indiana State Standards:

- 7.1.4 Understand and compute whole number powers of whole numbers.
- 7.3.9 Identify functions as linear or nonlinear and examine their characteristics in tables, graphs, and equations.
- 8.3.3 Interpret positive integer powers as repeated multiplication and negative integer powers as repeated division or multiplication by the multiplicative inverse.
- 8.3.8 Demonstrate an understanding of the relationships among tables, equations, verbal expressions, and graphs of linear functions.

The Problem:

How much is your time worth?

Would you rather work seven days at \$20 per day or be paid \$2 for the first day and have your salary double every day for a week?

*If the payment methods described in the “Challenge” were carried out for a month, how much money would you have earned altogether on the 30th day?**

Mathematical Content:

This task requires students to determine a method for finding the total amount of money earned when paid double each day. Students must then find a more generalized method for finding this figure, by utilizing their knowledge about patterns, and apply it to the second part of the problem.

Solution:

If you are paid \$20 per day for seven days, then you earn $\$20 \cdot 7 = \140 . If you are paid \$2 for the first day and your salary doubles every day for the next six days, then you earn $\$2 + \$4 + \$8 + \$16 + \$32 + \$64 + \$128 = \254 . The second scheme earns you more money by the end of the week.

If the payment methods were carried out for a month you would earn \$600 by the first method and \$2,147,483,646 by the second!

* Source: National Council of Teachers of Mathematics. (2004). Math Challenge #7, Double or Not. *Figure This!* Retrieved August 8, 2005, from <http://www.figurethis.org/challenges/c07/challenge.htm>

Algebra 6-8:

- Model and solve contextualized problems using various representations, such as graphs, tables, and equations.
- Use graphs to analyze the nature of changes in quantities in linear relationships.

Indiana State Standards:

- 6.3.7 Identify and graph ordered pairs in the four quadrants of the coordinate plane.
- 6.3.8 Solve problems involving linear functions with integer values. Write the equation and graph the resulting ordered pairs of integers on a grid.
- 6.3.9 Investigate how a change in one variable relates to a change in a second variable.
- 7.3.7 Find the slope of a line from its graph.
- 7.3.8 Draw the graph of a line given the slope and one point on the line, or two points on the line.
- 7.3.9 Identify functions as linear or nonlinear and examine their characteristics in tables, graphs, and equations.
- 7.3.10 Identify and describe situations with constant or varying rates of change and know that a constant rate of change describes a linear function.
- 8.3.5 Identify and graph linear functions and identify lines with positive and negative slope.
- 8.3.6 Find the slope of a linear function given the equation and write the equation of a line given the slope and any point on the line.
- 8.3.7 Demonstrate an understanding of rate as a measure of one quantity with respect to another quantity.
- 8.3.8 Demonstrate an understanding of the relationships among tables, equations, verbal expressions, and graphs of linear functions.

The Problem:

Memory chips are fabricated on silicon wafers. Efforts are made to "shrink" the chip size so that more chips can be made per wafer. Following the fabrication process of the chips, the wafers are tested for functionality. Wafers are tested with voltages that stress their performance causing those devices that do not meet specifications to fail. "Yield" refers to the number or percentage of acceptable units (chips) produced on each wafer compared to the maximum possible.

Yield vs. Parameter

Use graph paper to create an x-y graph of yields vs. electrical parameters measured in volts.

Parameter	Yield
5v	50%
10v	60%
15v	70%

- How does the parameter relate to the yield? (Positive vs. negative correlation)*
- Where do we want the parameter to be set in order to obtain the highest yield?*
- What's the equation of the line?*
- We did more tests, raising the voltage for the parameter (the new data is on the following page). Plot both the old and new data. Now, where does the graph say to set the volts (to obtain the highest yield)?**

Old Data		New Data	
Parameter	Yield	Parameter	Yield
5v	50%	20v	80%
10v	60%	30v	30%
15v	70%	40v	20%

Mathematical Content:

This task asks students to demonstrate their knowledge of graphing, determining the equation of a line, and observing maximum values. First, students are asked to determine the type of correlation between two sets of data. Students must then determine the parameter for the highest yield of acceptable units on each wafer. Next, students must find the equation of the line produced by the data points given using the point-slope formula. Lastly, students must plot two sets of data on the same graph and determine the highest yield point in the new set of data.

Solution:

Graph of Yield vs. Electrical Parameters measured in volts.

Graphs may vary slightly.

Students should graph yield along y-axis and parameter on the x-axis.

- How does the parameter related to the yield?*
Positive correlation – As voltage increases, the yield percentage increases.
- Where do we want the parameter to be for the highest yield?*
15v would be the parameter that produces the highest yield.

* Source: Micron Technology, Inc. (2005). Math in the Workplace: Algebra Problems. *Math in the Workplace*. Retrieved August 15, 2005, from <http://www.micron.com/k12/math/algebra/yield2.html>

- c. *What's the equation of the line?*

Use the point-slope formula.

$$y - y_1 = m(x - x_1)$$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = 2$$

$$y - 50 = 2(x - 5)$$

$$y = 2x - 10 + 50$$

$$y = 2x + 40$$

- d. *We did more tests, raising the voltage for the parameter. Plot both the old and new data. Now, where does the graph say to set the volts?*

Graphs may vary slightly. Students should plot the old data and new data on the same graph. Based on the new data, the volts should be set at 20.

Algebra 6-8:

- relate and compare different forms of representation for a relationship
- identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations
- explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope

Indiana State Standards:

- 7.3.9 Identify functions as linear or nonlinear and examine their characteristics in tables, graphs, and equations.
- 8.3.8 Demonstrate an understanding of the relationships among tables, equations, verbal expressions, and graphs of linear functions.
- 8.3.9 Represent simple quadratic functions using verbal descriptions, tables, graphs, and formulas and translate among these representations.

The Problem:

*The function $A = 10x - x^2$, where x is the width in yards gives the area A in square yards. Graph the function. Use the graph to find the width that gives the greatest value.**

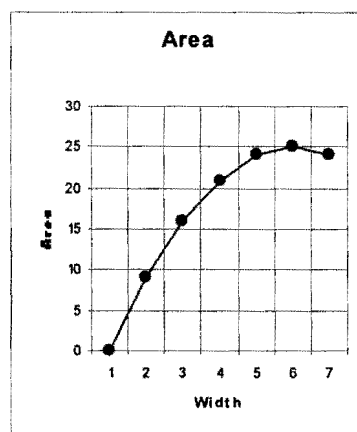
Mathematical Content:

This task asks students to graph a quadratic function. This activity might be best placed after a discussion on linear graphs. Students could use their knowledge about how to graph linear functions in order to graph this new type of function. There are several ways that students may graph this function, only one method for doing so is listed as a solution.

Solution:

Students may find it helpful to make a table and then graph the information from their table. Student graphs should resemble the one below.

x	$10x - x^2$	(x, y)
0	$10(0) - (0)^2$	(0, 0)
1	$10(1) - (1)^2$	(1, 9)
2	$10(2) - (2)^2$	(2, 16)
3	$10(3) - (3)^2$	(3, 21)
4	$10(4) - (4)^2$	(4, 24)
5	$10(5) - (5)^2$	(5, 25)
6	$10(6) - (6)^2$	(6, 24)



* Source: Charles, R., Branch-Boyd, J., Illingworth, M., Mills, D., & Reeves, A. (2004). *Prentice Hall Mathematics: Pre-Algebra*. Needham, MA: Pearson Education, Inc.

Algebra 9-12:

- Use symbolic algebra to represent and explain mathematical relationships

Indiana State Standards:

- 8.3.7 Demonstrate an understanding of rate as a measure of one quantity with respect to another quantity.
- A1.2.2 Solve equations and formulas for a specified variable.
- A1.2.6 Solve word problems that involve linear equations, formulas, and inequalities.
- A1.5.3 Understand and use the substitution method to solve a pair of linear equations in two variables.

The Problem:

*A flight from Chicago to Tokyo is scheduled to take 13 hours but departs 52 minutes late. How should the pilot adjust his average speed for the plane to arrive on time?**

Mathematical Content:

This task is one in which students should recall the formula $\text{rate} \times \text{time} = \text{distance}$. Students should recognize that the pilot travels the same distance each time, therefore the two equations they obtain by applying the formula can be set equal to one another. To determine the rate at which the pilot should travel as compared to the rate he traveled on the 13 hr flight, students must solve for the a specific variable. Upon solving for the variable, students should recognize the difference in rates needed for the pilot to arrive at the same time.

Solution:

Increase speed by $\frac{1}{14}$ of the original. Given that the distance is the same, $r_1 t_1 = r_2 t_2$.

$$t_1 = 13 \text{ hours}$$

$$= 780 \text{ mins}$$

$$t_2 = 13 \text{ hours} - 52 \text{ mins}$$

$$= 728 \text{ mins}$$

* Source: McGivney-Burelle, J. & White, J. (2005). May Calendar. *Mathematics Teacher*, 98 (9), 616.

So,

$$r_1(780) = r_2(728)$$

$$r_2 = \frac{780}{728} r_1$$

$$r_2 = \frac{15}{14} r_1$$

$$r_2 = \left(1 + \frac{1}{14}\right) r_1$$

or $r_2 = \frac{1}{14}$ more than the original rate, r_1 .

Algebra 9-12:

- Use symbolic algebra to represent and explain mathematical relationships

Indiana State Standards:

8.3.7 Demonstrate an understanding of rate as a measure of one quantity with respect to another quantity.

A1.2.2 Solve equations and formulas for a specified variable.

A1.2.6 Solve word problems that involve linear equations, formulas, and inequalities.

A1.5.3 Understand and use the substitution method to solve a pair of linear equations in two variables.

The Problem:

*Mr. Earl E. Bird leaves his house for work at exactly 8:00 a.m. every morning. When he averages 20 miles per hour, he arrives at his workplace three minutes late. When he averages 60 miles per hour, he arrives 3 minutes early. At what average speed, in miles per hour, should Mr. Bird drive to arrive at his workplace precisely on time?**

Mathematical Content:

In this task students are asked to calculate the speed Mr. Bird must travel in order to arrive at work exactly on time. In order to complete this task, students should recall the formula $\text{rate} \times \text{time} = \text{distance}$. Because Mr. Bird is traveling the same distance on each occasion, the student should see that the information in the problem can be written as two equations set equal to one another. After doing so, this task is reduced to solving for a single variable and using substitution to determine the distance traveled. Upon finding the distance Mr. Bird travels students can determine the average speed Mr. Bird must travel per hour in order to arrive at his destination at precisely 8:00 a.m.

Solution:

48 miles per hour.

Let t be the number of hours that Mr. Bird must travel to arrive on time. Since three minutes is the same as 0.05 hours, $40(t + 0.05) = 60(t - 0.05)$. Thus, $40t + 2 = 60t - 3$, so $t = 0.25$.

The distance from Mr. Bird's home to work is $40(0.25 + 0.05)$ or 12 miles. Therefore, his average speed should be $\frac{12}{.25}$, or 48 miles per hour.

* Source: Covington, J. (2004). September Calendar. *Mathematics Teacher*, 98 (2), 104.

Algebra 9-12:

- Generalize patterns using explicitly defined and recursively defined functions

Indiana State Standards:

A1.2.1 Solve linear equations.

A1.2.2 Solve equations and formulas for a specified variable.

A1.2.6 Solve word problems that involve linear equations, formulas, and inequalities.

The Problem:

*Jenny attends basketball practice every day after school. At each practice last week, she made twice as many free throws as she had made at the previous practice. At her fifth practice she made 48 free throws. How many total free throws did she make during the week?**

Mathematical Content:

In this task students must utilize the information given in the problem to determine the pattern in which Jenny is making free throws. Once students have determined the pattern, they can easily work backwards to determine the amount of free throws Jenny made during each practice. After finding the amount of free throws made at each practice, students should recognize that it takes only simple addition to determine the amount of free throws Jenny made during the week.

Solution:

93. At Jenny's fourth practice she made $\left(\frac{1}{2}\right)(48) = 24$ free throws. She made 12 free throws at her third practice, 6 at her second practice, and 3 at her first practice. During the week, she had a total of $48 + 24 + 12 + 6 + 3 = 93$ free throws.

* Source: McGivney-Burelle, J. & White, J. (2005). August Calendar. *Mathematics Teacher*, 99 (1), 40.

Algebra 9-12:

- Use symbolic algebra to represent and explain mathematical relationships

Indiana State Standards:

A1.2.2 Solve equations and formulas for a specified variable.

A1.2.6 Solve word problems that involve linear equations, formulas, and inequalities.

A1.5.3 Understand and use the substitution method to solve a pair of linear equations in two variables.

A1.5.6 Use pairs of linear equations to solve word problems.

A2.2.2 Use substitution, elimination, and matrices to solve systems of two or three linear equations in two or three variables.

A2.2.3 Use systems of linear equations and inequalities to solve word problems.

The Problem:

*Patty has 20 coins consisting of nickels and dimes. If her nickels were dimes and her dimes were nickels, her coins would be worth 70 cents more. How much are her coins worth?**

Mathematical Content:

This task requires students to demonstrate their ability to create algebraic equations and use substitution to solve for a certain variable. Students could also find a solution to this problem by thinking critically about what it means that the values of Patty's coins were switched.

Solution:

From the problem statement, we know that the total amount of coins is 20. Therefore, $n + d = 20$, $n = \#$ of nickels, $d = \#$ of dimes. We also know that if the nickels and dimes were switched the total worth of the coins would increase by 70 cents. Which provides the equation: $.1n + .05d = .1d + .05n + .70$ OR $.05n - .05d = .70$.

Solving the initial equation for d yields $d = 20 - n$.

We can then use substitution to solve the second equation for n to determine the number of nickels that Patty has.

$$.05n - .05(20 - n) = .70$$

$$.05n - .05(20) + .05n = .70$$

$$.1n - 1 = .70$$

$$.1n = 1.70$$

$$n = 17$$

Therefore, there are 17 nickels. And, using the equation $d = 20 - n$ we find that there are $d = 20 - 17 = 3$ dimes. To determine the amount Patty's coins are worth, we simply add the values of the coins to obtain $.10(3) + .05(17) = \$1.15$.

* Source: McGivney-Burelle, J. & White, J. (2005). September Calendar. *Mathematics Teacher*, 99 (2), 112.

Additional Solution:

\$1.15. Because the value of Patty's money would increase if the dimes and nickels were interchanged, she must have more nickels than dimes. Interchanging one nickel for a dime increases the amount by 5 cents, so she has $\frac{70}{5} = 14$ more nickels than dimes.

Therefore, she has $\left(\frac{1}{2}\right)(20 - 14) = 3$ dimes and $20 - 3 = 17$ nickels.

MEASUREMENT PROBLEMS

MEASUREMENT RUBRIC

CATEGORY	1	2	3	4
Mathematical Concepts	Explanation shows very limited understanding of the concepts involved OR is not written. Student does not meet measurement expectations or standards.	Explanation shows some understanding of the concepts needed to solve the problem. Student may partially meet standards and expectations addressed in the problem.	Explanation shows substantial understanding of the concepts used to solve the problem. Student demonstrates adequate fulfillment of expectations and meets standards.	Explanation shows complete understanding of the concepts used to solve the problem. Student demonstrates fulfillment of expectations and meets standards.
Mathematical Errors	More than 75% of the steps and solutions have mathematical errors.	Most (75-84%) of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solutions have no mathematical errors.	90-100% of the steps and solutions have no errors.
Explanation	Explanation is difficult to understand and is missing several components OR was not included.	Explanation is a little difficult to understand, but includes critical components.	Explanation is clear.	Explanation is detailed and clear.
Mathematical Terminology and Notation	There is little use, or a lot of inappropriate use, of terminology and notation. No use of measurement labels.	Correct terminology and notation is used, but it is sometimes not easy to understand what was done. Measurement labels are not used or are incorrectly used.	Correct terminology and notation are usually used, making it fairly easy to understand what was done. Measurement labels are used correctly but inconsistently.	Correct terminology and notation are always used, making it easy to understand what was done. Measurement labels are used correctly.
Neatness and Organization	The work appears sloppy and unorganized. It is hard to determine what information goes together.	The work is presented in an organized fashion but may be hard to read at times.	The work is presented in a neat and organized fashion that is usually easy to read.	The work is presented in a neat, clear, organized fashion that is always easy to read.

Measurement 6-8:

- Understand both metric and customary systems of measurement
- Understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume
- Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision
- Develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders

Indiana State Standards:

- 6.5.1 Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles.
- 6.5.8 Use strategies to find the surface area and volume of right prisms and cylinders using appropriate units.
- 7.5.1 Compare lengths, areas, volumes, weights, capacities, times, and temperatures within measurement systems.
- 7.5.4 Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms, trapezoids, triangles, circles, right prisms, and cylinders.
- 8.5.4 Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms, trapezoids, triangles, circles, prisms, cylinders, spheres, cones, and pyramids.

The Problem:

What's a fair share?

Ratio and five friends want to share a 9 inch square chocolate cake with marshmallow icing. How can ratio cut the cake so that each person receives an equal share of both cake and icing?

*Hint: If six people will share the whole cake, then three people will share half the cake. Don't forget the icing on the sides!**

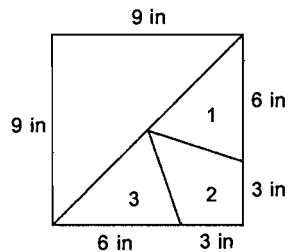
Mathematical Content:

This task requires students to visualize a three dimensional object. Students must use their knowledge of perimeter, surface area, and volume to be sure that each person at the party receives the same amount of icing and cake.

* Source: National Council of Teachers of Mathematics. (2004). Math Challenge #54, Cut the Cake. *Figure This!* Retrieved August 8, 2005 from <http://www.figurethis.org/challenges/c54/challenge.htm>

Solution:

There are many ways to approach this problem. Assuming that all pieces of the cake have the same height, the size (or volume) of each piece depends on the area of its top. Since icing is on both the top and sides, however, each piece must also have an equal share of the perimeter of the square. If six people share the whole cake, then three people will share half the cake. One way to divide the cake in half is to cut it along the diagonal of the square. Each half of the cake is 18 inches on its two outer edges. So each person should receive $18/3$, or 6 inches, of the cake's outer edge. One way to make this division is shown below.



To check the areas of the three pieces, use the formula for the area of a triangle:

$$Area = \frac{1}{2} \text{base} \cdot \text{height}$$

The height of the triangles 1 and 3 in the diagram is half the width of the cake, or 4.5 inches. Since the length of each base is 6 inches, the area of each of these two triangles is: $A = \frac{1}{2} \cdot 6 \cdot 4.5 = 13.5$ sq. in.

The area of shape 2 is half the area of the entire cake, minus the area of triangles 1 and 3:

$$Area \text{ of half} = \frac{1}{2} \cdot 9 \cdot 9 = \frac{81}{2} = 40.5, \text{ or } 40.5 \text{ sq. in.}$$

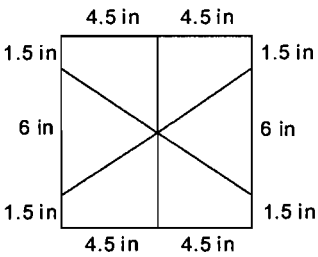
$$Area_{\text{Triangle 1}} = Area_{\text{Triangle 3}} = \frac{1}{2} \cdot 6 \cdot 4.5 = 13.5, \text{ or } 13.5 \text{ sq. in.}$$

$$Area_{\text{region 2}} = Area \text{ of half of the cake} - Area_{\text{Triangle 1}} - Area_{\text{Triangle 2}}$$

$$Area_{\text{region 2}} = \frac{1}{2} \cdot 9 \cdot 9 - 13.5 - 13.5 = 13.5, \text{ or } 13.5 \text{ sq. in.}$$

Therefore, these three pieces represent equal shares. The other half of the cake can be divided similarly.

There are other ways to cut the cake into six equal shares. For example, you can start by dividing the cake into two rectangles of the same size and shape. A cake cut in this manner is shown on the following page.



Measurement 6-8:

- Understand both metric and customary systems of measurement
- Understand relationships among units and convert from one unit to another within the same system
- Solve simple problems involving rates and derived measurements for such attributes as velocity and density

Indiana State Standards:

- 6.5.1 Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles.
- 6.5.2 Understand and use larger units for measuring length by comparing miles to yards and kilometers to meters.
- 7.5.1 Compare lengths, areas, volumes, weights, capacities, times, and temperatures within measurement systems.
- 8.5.1 Convert common measurements for length, area, volume, weight, capacity, and time to equivalent measurements within the same system.
- 8.5.2 Solve simple problems involving rates and derived measurements for attributes such as velocity and density.

The Problem:

Can you run as fast as a car?

*During the 100 meter dash in the 1988 Olympic Games in Seoul, Florence Griffith-Joyner was timed at 0.91 seconds for 10 meters. At that speed, could she pass a car traveling 15 miles per hour in a school zone?**

Mathematical Content:

This task requires students to use their knowledge of the standard measurement system. Students may use unit analysis in order to convert Florence's time from seconds per 10 meters to miles per hour. Doing so means students must be familiar with cross cancellation and various units of measurement.

Solution:

One method for converting between measures is called dimensional analysis. The conversions between measures are written as fractions so the common units cancel out.

$$\frac{10m}{0.91\text{sec}} \cdot \frac{100cm}{1m} \cdot \frac{1in}{2.54cm} \cdot \frac{1ft}{12in} \cdot \frac{1mi}{5280ft} \cdot \frac{60\text{sec}}{1\text{min}} \cdot \frac{60\text{min}}{1hr} = \frac{24.6mi}{hr}$$

Her rate is about 24.5 miles per hour, and she could easily pass a car going a rate of 15 miles per hour.

* Source: National Council of Teachers of Mathematics. (2004). Math Challenge #34, Faster? *Figure This!* Retrieved August 8, 2005 from <http://www.figurethis.org/challenges/c34/challenge.htm>

Measurement 6-8:

- Understand both metric and customary units of measurement
- Solve simple problems involving rates and derived measurements for such attributes as velocity and density

Indiana State Standards:

- 6.5.1 Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles.
- 7.5.1 Compare lengths, areas, volumes, weights, capacities, times, and temperatures within measurement systems.
- 7.5.2 Read and create drawings made to scale, construct scale models, and solve problems related to scale.
- 8.5.3 Solve simple problems involving rates and derived measurements for attributes such as velocity and density.

The Problem:

Who runs faster?

*When Polygon and Exponent ran a 50 meter race, Polygon crossed the finish line while Exponent was at the 45-meter mark. The two friends decide to race again. This time, Exponent starts 5 meters ahead of Polygon, who is at the original starting line. If each runs at the same speed as in the previous race, who will win?**

Mathematical Content:

This task requires students to visualize the given situation. Students must “see” what it means for Polygon to run the full length of the race, while Exponent does not. Students must also develop an understanding of what it means to run at the same speed and how that affects the amount of distance covered.

Solution:

In the first race, Exponent ran 45 meters in the same time it took Polygon to run 50 meters. In the second race, the finish line is 45 meters from Exponent’s starting position and 50 meters from Polygon’s. Therefore, if each runs at the same speed as in the previous race, the race will end in a tie. See the diagram on the following page.

* Source: National Council of Teachers of Mathematics. (2004). Math Challenge #74, The Race. *Figure This!* Retrieved August 8, 2005 from <http://www.figurethis.org/challenges/c74/challenge.htm>

Start	<u>50</u>	Polygon
Start	<u>45</u>	Exponent
Start	<u>50</u>	Polygon
<u>5</u> Start	<u>50</u>	Exponent

Measurement 6-8:

- Understand both metric and customary systems of measurement
- Use common bench marks to select appropriate methods for estimating measurements
- Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision

Indiana State Standards:

6.5.1 Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles.

7.5.1 Compare lengths, areas, volumes, weights, capacities, times, and temperatures within measurement systems.

The Problem:

*Coins in the United States have the following thicknesses: penny, 1.55 mm; nickel, 1.95 mm; dime, 1.35 mm; quarter, 1.75 mm. If a stack of coins is exactly 14 mm high, how many coins are in the stack?**

Mathematical Content:

This task requires students to think critically about the given situation. One way students might solve this problem is to simply guess and check and find a solution that works. However, the problem is much simpler if students analyze what it means for the stack to add up to 14.00 mm. For instance, students might think about what it means for the coins to be added together, what kind of number do they get in the tenths and hundredths places?

Solution:

8. The height in millimeters of any stack with an odd number of coins would have a 5 in the hundredths place. The height of any two coins would have an odd digit in the tenths place and a zero in the hundredths place. Therefore any stack with zeros in both its tenths and hundredths places must consist of a number of coins that is a multiple of 4. The highest stack of 4 coins would only have a height of $4(1.95) = 7.8 \text{ mm}$, which is too short. The shortest stack of 12 coins would have a height of $12(1.35) = 16.2 \text{ mm}$, which is too tall. This indicates that the only possible multiple of 4 that will work is a stack of 8 coins. Note that a stack of 8 quarters has height of $8(1.75) = 14 \text{ mm}$.

* Source: McGivney-Burelle, J. & White, J. (2005). August Calendar. *Mathematics Teacher*, 99 (1), 40.

Measurement 6-8:

- Understand both metric and customary systems of measurement
- Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision

Indiana State Standards:

- 6.5.1 Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles.
- 7.5.1 Compare lengths, areas, volumes, weights, capacities, times, and temperatures within measurement systems.
- 7.5.2 Read and create drawings made to scale, construct scale models, and solve problems related to scale.
- 7.5.3 Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms, trapezoids, triangles, circles, right prisms, and cylinders.
- 8.5.4 Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms, trapezoids, triangles, circles, prisms, cylinders, spheres, cones, and pyramids.

The Problem:

*Minneapolis-St. Paul International Airport is 8 miles southwest of downtown St. Paul and 10 miles southeast of downtown Minneapolis. To the nearest integer, what is the number of miles between downtown St. Paul and downtown Minneapolis?**

Mathematical Content:

This task requires students to visualize the given situation. In order to find the solution to this problem, students must first draw an accurate picture of the situation described. Students must then refer to their knowledge of geometry and navigational directions in order to find the distance between St. Paul and downtown Minneapolis.

Solution:

13 miles. Let downtown St. Paul, downtown Minneapolis, and the airport be located at S , M , and A , respectively. Then $\triangle MAS$ has a right angle at A , so by the Pythagorean Theorem, $MS = \sqrt{10^2 + 8^2} = \sqrt{164} \approx \sqrt{169}$, or 13.

* Source: McGivney-Burelle, J. & White, J. (2005). August Calendar. *Mathematics Teacher*, 99 (1), 40.

Measurement 6-8:

- Understand both metric and customary systems of measurement
- Understand relationships among units and convert from one unit to another within the same system
- Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision

Indiana State Standards:

- 6.5.1 Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles.
- 7.5.1 Compare lengths, areas, volumes, weights, capacities, times, and temperatures within measurement systems.
- 7.5.4 Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms, trapezoids, triangles, circles, right prisms, and cylinders.
- 8.5.1 Convert common measurements for length, area, volume, weight, capacity, and time to equivalent measurements within the same system.
- 8.5.5 Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms, trapezoids, triangles, circles, prisms, cylinders, spheres, cones, and pyramids.

The Problem:

The cost of carpet is \$13.00/square yard and the cost to install the carpet is \$5.00/square yard. The cost of hardwood flooring is \$7.00/square foot and the installation cost is \$2.00/square foot.

Jon's family room measures 18 ft by 24 ft.

What is the cost for carpet, including installation?

What is the cost for hardwood flooring, including installation?

*What is the total cost for hardwood including installation, around a perimeter four feet wide; and carpet, including installation, in the remaining middle area of the family room?**

* Source: Micron Technology, Inc. (2005). Math in the Workplace: Geometry Problems. *Math in the Workplace*. Retrieved August 15, 2005, from <http://www.micron.com/k12/math/geometry/prodmgr1.html>

Mathematical Content:

This problem requires students to think critically about what the problem is asking for. The problem is broken down into three specific questions, each of which requires a different level of knowledge about measurement. The first question is relatively simple, requiring students to merely find the area of the space to be carpeted and then find the amount that Jon must spend in order to carpet that area using some simple unit transformations. The second question requires students to merely find the cost of hardwood given the price in the same units as the area. The last question is more complicated but utilizes the processes described for finding solutions to the first two questions. This problem draws on the knowledge of transformation between metric and customary unit measurements.

Solution:

What is the cost for carpet including installation?

$$24 \text{ ft} \times 18 \text{ ft} = 432 \text{ ft}^2 \text{ of carpet needed}$$

$$\$13 + \$5 = \$18 / \text{yd}^2 \text{ installed carpet}$$

$$\frac{432 \text{ ft}^2}{9 \text{ ft}^2} = 48 \text{ yd}^2 \times \$18 / \text{yd}^2 = \$864 \text{ for carpet}$$

What is the cost of hardwood flooring including installation?

$$432 \text{ ft}^2 \times \$9 / \text{ft}^2 = \$3,888 \text{ for hardwood floor}$$

What is the total cost for the hardwood and carpet layout as described?

$$(24 \text{ ft} \times 18 \text{ ft}) - (16 \text{ ft} \times 10 \text{ ft}) = 272 \text{ ft}^2 \text{ of hardwood needed}$$

$$\$7 + \$2 = \$9 / \text{ft}^2 \text{ installed hardwood}$$

$$272 \text{ ft}^2 \times \$9 / \text{ft}^2 = \$2,448 \text{ for the 4 ft hardwood perimeter}$$

$$\frac{16 \text{ ft} \times 10 \text{ ft}}{9 \text{ ft}^2} = 17.77 \text{ yd}^2 \text{ of carpet needed}$$

$$17.77 \text{ yd}^2 \times \$18 / \text{yd}^2 = \$320 \text{ for carpet}$$

$$\$2,448 + \$320 = \$2,768 \text{ total for hardwood and carpet}$$

Measurement 6-8:

- Understand both metric and customary systems of measurement
- Use common benchmarks to select appropriate methods for estimating measurements
- Solve simple problems involving rates and derived measurements for such attributes as velocity and density

Indiana State Standards:

- 6.5.1 Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles.
- 7.5.1 Compare lengths, areas, volumes, weights, capacities, times, and temperatures within measurement systems.
- 8.5.1 Convert common measurements for length, area, volume, weight, capacity, and time to equivalent measurements within the same system.
- 8.5.2 Solve simple problems involving rates and derived measurements for attributes such as velocity and density.
- 8.5.3 Solve problems involving scale factors, area, and volume using ratio and proportion.

The Problem:

Which tastes juicier?

If all grape juice concentrates are the same strength, which recipe would you expect to have the strongest grape taste?

<u>Recipe</u>	<u>Concentrate</u>	<u>Water</u>
Jerry's Juice	2 cups concentrate	3 cups water
Grapeade	5 cups concentrate	8 cups water
Good Grape	3 cups concentrate	4 cups water
Jane's Juice	4 cups concentrate	7 cups water

*Hint: for each recipe think about how much water should be used with 1 cup (c.) of concentrate, or how much concentrate should be used with 1 cup of water.**

Mathematical Content:

This task forces students to analyze the strength of concentrate by using proportions. Students may use unit conversion to determine the amount of concentrate per cup of water or the amount of water per cup of concentrate.

* Source: National Council of Teachers of Mathematics. (2004). Math Challenge #25, Grape Juice Jungle. *Figure This!* Retrieved August 8, 2005, from <http://www.figurethis.org/challenges/c25/challenge.htm>

Solution:

There are several ways to approach this problem.

One way is to determine how much concentrate each recipe uses for 1 cup of water. The one that uses the most concentrate should have the strongest grape taste.

Recipe	Cups of Concentrate per Recipe	Cups of Water per Recipe	Ratio of Concentrate to Water	Ratio of Concentrate to 1 cup of Water
Jerry's Juice	2	3	$\frac{2}{3}$	$\frac{2 \div 3}{3 \div 3} \approx \frac{0.67}{1}$
Grapeade	5	8	$\frac{5}{8}$	$\frac{5 \div 8}{8 \div 8} \approx \frac{0.63}{1}$
Good Grape	3	4	$\frac{3}{4}$	$\frac{3 \div 4}{4 \div 4} = \frac{0.75}{1}$
Jane's Juice	4	7	$\frac{4}{7}$	$\frac{4 \div 7}{7 \div 7} \approx \frac{0.56}{1}$

Good Grape has the most concentrate (0.75) for 1 cup of water. It should have the strongest grape taste.

Another way is to find how much water each recipe uses for 1 cup of concentrate. Here, the recipe that uses the least water should have the strongest grape taste.

Recipe	Cups of Concentrate per Recipe	Cups of Water per Recipe	Ratio of Concentrate to Water	Ratio of Concentrate to 1 cup of Water
Jerry's Juice	2	3	$\frac{2}{3}$	$\frac{2 \div 2}{3 \div 2} = \frac{1}{1.5}$
Grapeade	5	8	$\frac{5}{8}$	$\frac{5 \div 5}{8 \div 5} = \frac{1}{1.6}$
Good Grape	3	4	$\frac{3}{4}$	$\frac{3 \div 3}{4 \div 3} \approx \frac{1}{1.3}$
Jane's Juice	4	7	$\frac{4}{7}$	$\frac{4 \div 4}{7 \div 4} \approx \frac{1}{1.8}$

Good Grape has the least amount of water, 1.3 cups, to 1 cup of concentrate and so should have the most grape flavor.

Measurement 6-8:

- Understand both metric and customary systems of measurement
- Understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume
- Use common benchmarks to select appropriate methods for estimating measurements
- Develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and develop strategies to find the area of more-complex shapes

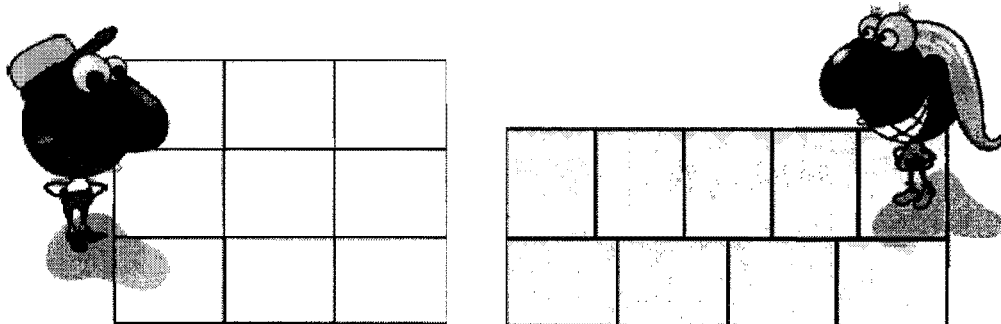
Indiana State Standards:

- 6.5.1 Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles.
- 7.5.5 Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms, trapezoids, triangles, circles, right prisms, and cylinders.
- 7.5.6 Estimate and compute the area of more complex or irregular two-dimensional shapes by dividing them into more basic shapes.
- 8.5.4 Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms, trapezoids, triangles, circles, prisms, cylinders, spheres, cones, and pyramids.

The Problem:

Does bigger perimeter mean bigger area?

*Helix and Polygon both used the same number of identical concrete pieces to make their patios. The area of each patio is the same: 180 square meters. What are the dimensions of a single piece of concrete?**



* Source: National Council of Teachers of Mathematics. (2004). Math Challenge #39, Patios. *Figure This!* Retrieved August 8, 2005, from <http://www.figurethis.org/challenges/c39/challenge.htm>

Hint: Notice how the pieces fit together on Polygon's patio. What is different about the way the pieces fit on Helix's patio?

Mathematical Content:

This task requires students to think critically about the concepts of area and perimeter. Students must determine the dimensions for a single piece of concrete given various bits of information. In this task, students must consider the drawings and what information they provide that might be of help.

Solution:

The area of each patio is 180 square meters (m^2), and each is made from nine identical rectangular concrete pieces. This means that the area of one piece is $180 \div 9 = 20 \text{ } m^2$. Since the area of the rectangle equals length (l) times width (w), you know that $l \cdot w = 20$. From the way in which the pieces are arranged in Polygon's patio, you can see that four lengths is the same as five widths. In other words, the ratio of length to width is 5 to 4. As it happens, this means that the length is 5 m , while the width is 4 m .

Another way to look at this is as follows. You know from Polygon's patio that four lengths equals five widths. Since $4l = 5w$, then:

$$l = \frac{5}{4}w$$

You also know the area of the piece: $l \cdot w = 20$. By substituting for l .

$$\left(\frac{5}{4}w\right) \cdot w = 20$$

$$\left(\frac{5}{4}\right)w^2 = 20$$

$$w^2 = \frac{4}{5} \cdot 20$$

$$w^2 = 16$$

$$w = 4 \text{ (widths cannot be negative)}$$

Substituting 4 for w , you can then determine that $l = 5$. So a single concrete piece has dimensions 4 m by 5 m .

Measurement 6-8:

- Understand both metric and customary systems of measurement
- Understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume
- Use common benchmarks to select appropriate methods for estimating measurements
- Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision
- Develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more-complex shapes

Indiana State Standards:

- 6.5.1 Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles.
- 6.5.4 Understand the concept of the constant π as the ratio of the circumference to the diameter of a circle. Develop and use the formulas for the circumference and area of a circle.
- 7.5.1 Compare lengths, areas, volumes, weights, capacities, times, and temperatures within measurement systems.
- 7.5.4 Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms, trapezoids, triangles, circles, right prisms, and cylinders.
- 8.5.1 Convert common measurements for length, area, volume, weight, capacity, and time to equivalent measurements within the same system.
- 8.5.4 Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms, trapezoids, triangles, circles, prisms, cylinders, spheres, cones, and pyramids.

The Problem:

Have you ever seen a tree big enough to drive a car through?

*Are any of the “National Champion” trees in the table (on the following page) wide enough for a car to drive through?**

* Source: National Council of Teachers of Mathematics. (2004). Math Challenge #15, Big Trees. *Figure This!* Retrieved August 8, 2005, from <http://www.figurethis.org/challenges/c15/challenge.htm>

<i>Tree</i>	<i>Girth 4.5 ft Above Ground In Inches</i>	<i>Height in Feet</i>	<i>Location</i>
<i>American Beech</i>	<i>279</i>	<i>115</i>	<i>Harwood, MD</i>
<i>Black Willow</i>	<i>400</i>	<i>76</i>	<i>Grand Traverse Co., MI</i>
<i>Coast Douglas Fir</i>	<i>438</i>	<i>329</i>	<i>Coos Count, OR</i>
<i>Coast Redwood</i>	<i>267</i>	<i>313</i>	<i>Prairie Creek Redwoods State Park, CA</i>
<i>Giant Sequoia</i>	<i>998</i>	<i>275</i>	<i>Sequoia National Park, CA</i>
<i>Loblolly Pine</i>	<i>188</i>	<i>148</i>	<i>Warren, AZ</i>
<i>Pinyon Pine</i>	<i>213</i>	<i>69</i>	<i>Cuba, NM</i>
<i>Sugar Maple</i>	<i>274</i>	<i>65</i>	<i>Kitzmilller, MD</i>
<i>Sugar Pine</i>	<i>442</i>	<i>232</i>	<i>Dorrington, CA</i>
<i>White Oak</i>	<i>382</i>	<i>96</i>	<i>Wye Mills State Park, MD</i>

Hint: The distance around a tree is its girth. You may also want to estimate the width of a car at approximately 6 ft.

Mathematical Content:

This task requires students to convert from inches to feet or vice versa. Once students have converted the measurements into their length of choice, students must determine the diameter of the trees in the table given their circumference. Ultimately, students must decide which trees have a diameter that is wide enough for a car to pass through.

Solution:

To find the diameter of a circle when you know the circumference, you divide the circumference by π (about 3.14). For example, the Black Willow has a girth of 400 inches and since $400 \div 3.14 \approx 127$, the Black Willow is about 127 inches wide or $127 \div 12 \approx 11$ ft, enough for a car to drive through and still leave at least 2 ft on each side. See the chart on the following page.

Tree	Girth 4.5 ft Above Ground In Inches	Diameter (Girth/ π) In Inches*	Diameter in Feet*
American Beech	279	89	7
Black Willow	400	127	11
Coast Douglas Fir	438	139	12
Coast Redwood	267	276	23
Giant Sequoia	998	318	27
Loblolly Pine	188	60	5
Pinyon Pine	213	68	6
Sugar Maple	274	87	7
Sugar Pine	442	141	12
White Oak	382	122	10

*Measurements rounded to the nearest whole number

The trees wide enough to drive a car through include: the Black Willow, Coast Douglas Fir, Coast Redwood, Giant Sequoia, and Sugar Pine.

Measurement 6-8:

- Understand both metric and customary systems of measurement
- Understand relationships among units and convert from one unit to another within the same system
- Solve simple problems involving rates and derived measurements for such attributes as velocity and density

Indiana State Standards:

- 6.5.1 Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles.
- 7.5.1 Compare lengths, areas, volumes, weights, capacities, times, and temperatures within measurement systems.
- 8.5.1 Convert common measurements for length, area, volume, weight, capacity, and time to equivalent measurements within the same system.
- 8.5.2 Solve simple problems involving rates and derived measurements for attributes such as velocity and density.

The Problem:

*How far can you go on a tank of gas? The Environmental Protection Agency (EPA) estimates for gas mileage on 1999 cars vary widely. Which of these cars should go the farthest on one tank of gas?**

<i>CENTURY</i>	<i>Metro</i>	<i>Navigator</i>
<i>20 / 29 mpg</i>	<i>44 / 49 mpg</i>	<i>13 / 17 mpg</i>
<i>City Highway</i>	<i>City Highway</i>	<i>City Highway</i>
<i>17.5 gal. Tank</i>	<i>10.3 gal. Tank</i>	<i>30.0 gal. Tank</i>

Hint: Suppose you only drive on the city. On the highway.

Mathematical Content:

This task requires students to develop a method for comparing the distance the given cars can go on one tank of gas. Students may use various combinations of highway and city miles to determine which car will go the farthest in that particular situation. Ultimately, this task forces students to recognize that it is the combination of highway and city miles that will determine which car will go the farthest.

Solution:

Different combinations of city and highway driving are possible. If you assume all driving is done on the highway, the Navigator should go the farthest as seen in the chart on the following page. But all three go approximately the same distance.

* Source: National Council of Teachers of Mathematics. (2004). Math Challenge #24, Gasoline Tanks. *Figure This!* Retrieved August 8, 2005, from <http://www.figurethis.org/challenges/c24/challenge.htm>

CAR	Highway Mileage (mpg)	Tank Capacity (gallons)	Approximate Distance (miles)
Century	29	17.5	508
Metro	49	10.3	505
Navigator	17	30	510

If you assume all driving is done in the city, the Metro will go the farthest.

Car	City Mileage (mpg)	Tank Capacity (gallons)	Approximate Distance (miles)
Century	20	17.5	350
Metro	44	10.3	453
Navigator	13	30	390

You might also consider different combinations of highway and city driving.

Measurement 6-8:

- Understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume
- Develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders

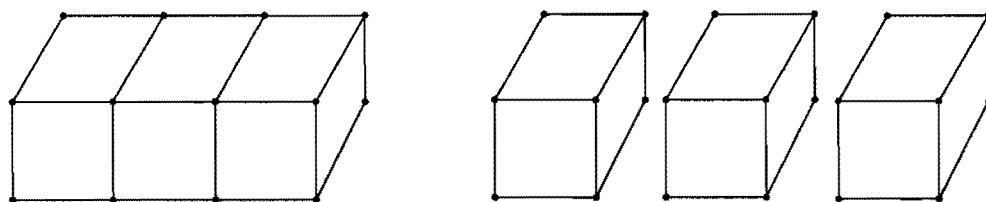
Indiana State Standards:

- 6.5.8 Use strategies to find the surface area and volume of right prisms and cylinders using appropriate units.
- 7.5.4 Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms, trapezoids, triangles, circles, right prisms, and cylinders.
- 7.5.6 Use objects and geometry modeling tools to compute the surface area of the faces and the volume of a three-dimensional object built from rectangular solids.
- 8.5.4 Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms, trapezoids, triangles, circles, prisms, cylinders, spheres, cones, and pyramids.

The Problem:

When should you buy block ice or crushed ice?

*Which typically melts faster, a single block of ice or the same block cut into three cubes?**



Hint: Compare the exposed areas.

Mathematical Content:

This task requires students to think critically about what it means for ice to melt. How can one determine which block of ice will melt faster? Students must determine what factors play a role in the melting of ice. Students should recognize that the amount of surface area exposed to heat plays a major role in the time it takes for an ice cube to melt.

* Source: National Council of Teachers of Mathematics. (2004). Math Challenge #62, Keeping Cool. *Figure This!* Retrieved August 8, 2005, from <http://www.figurethis.org/challenges/c62/challenge.htm>

Once students recognize that they must determine the surface area of the given objects in each situation the problem can easily be completed.

Solution:

Suppose that each of the smaller cubes is 1 inch high. The area of one face of the cube is length times width, $1 \cdot 1 = 1$ or 1 sq. in.

On each cube there are six faces, each with an area of 1 sq. in. The total surface area for each of the cubes is 6 sq in. The surface area of the three cubes is $3 \cdot 6 = 18$ or 18 sq. in. The big block of ice has four larger faces, each with an area of $3 \cdot 1$ for a total of 12 sq. in. The block also has two smaller faces with a surface area of $1 \cdot 1$, each, for a total of 2 sq. in. Thus, the surface area of the big block of ice is $12 + 2 = 14$ sq. in. The three cubes broken apart will melt more quickly since the surface area of 18 sq. in. is exposed while the block of ice has only 14 sq. in. exposed.

Measurement 6-8, 9-12:

- Understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume
- Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision
- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders

Indiana State Standards:

- 7.5.1 Compare lengths, areas, volumes, weights, capacities, times, and temperatures within measurement systems.
- 7.5.4 Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms*, trapezoids*, triangles, circles, right prisms*, and cylinders.
- 8.5.4 Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms*, trapezoids*, triangles, circles, prisms*, cylinders, spheres, cones, and pyramids.
- G.7.7 Find and use measures of sides, volumes of solids, and surface areas of solids. Relate these measures to each other using formulas.

The Problem:

*Given that they are made of the same material, which is heavier, a ball with a radius of 10 inches or 10 balls each with a radius of 1 inch?**

Mathematical Content:

This task requires students to use the formula for the volume of a sphere. The task is somewhat deceiving, but upon application of the proper formula students can easily determine which is heavier.

Solution:

The ball with a radius of 10 inches is heavier. Since the volume of a sphere is $\frac{4}{3}\pi r^3$,

one ball with a radius of 10 inches would have a volume of $\frac{4}{3}\pi \cdot 10^3 = \frac{4000}{3}\pi \text{ in}^3$. Thus,

10 balls of radius 1 inch would have an accumulated volume of only

$$10 \cdot \frac{4}{3}\pi 1^3 = \frac{40}{3}\pi \text{ in}^3.$$

* Source: McGivney-Burelle, J. & White, J. (2005). August Calendar. *Mathematics Teacher*, 99 (1), 40.

Measurement 6-8, 9-12:

- Understand both metric and customary systems of measurement
- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders

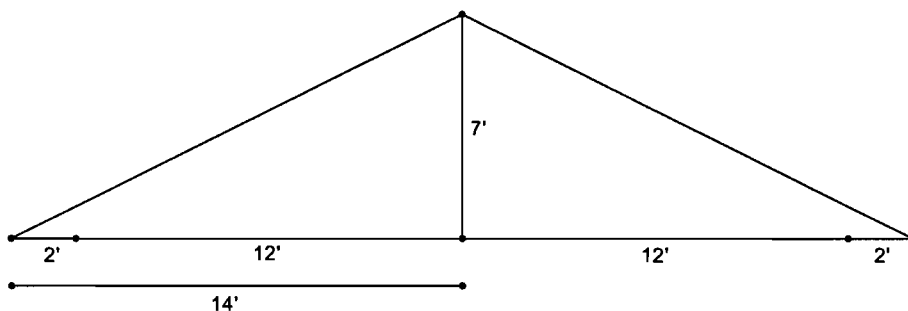
Indiana State Standards:

- 6.5.1 Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles.
- 7.4.3 Know and understand the Pythagorean Theorem and use it to find the length of the missing side of a right triangle and the lengths of other line segments. Use direct measurement to test conjectures about triangles.
- 7.5.4 Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms, trapezoids, triangles, circles, right prisms, and cylinders.
- 8.4.5 Use the Pythagorean Theorem and its converse to solve problems in two and three dimensions.
- 8.5.4 Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms, trapezoids, triangles, circles, prisms, cylinders, spheres, cones, and pyramids.
- G.3.3 Find and use measures of sides, perimeters, and areas of quadrilaterals. Relate these measures to each other using formulas.

The Problem:

Karen is figuring how many bundles of shingles to order for the roof on her garage. The garage is 24 ft by 24 ft, with a single roof ridge down the middle, plus a 2 ft overhang all around. The roof slope is 6" in 12".

*Three bundles of shingles are required for each 100 sq. ft. of roof. How many bundles of shingles should Karen order?**



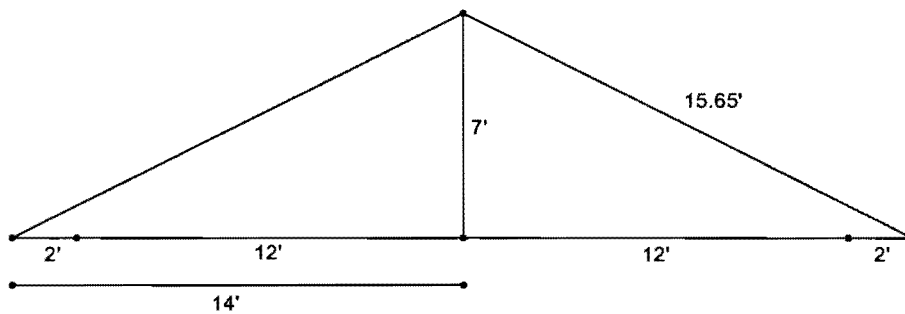
* Source: Micron Technology, Inc. (2005). Math in the Workplace: Geometry Problems. *Math in the Workplace*. Retrieved August 15, 2005, from <http://www.micron.com/k12/math/geometry/architect.html>

Mathematical Content:

This task requires students to use the Pythagorean Theorem to find the length of the missing edge of the roof as described. Using their solution students can determine the surface area of the roof. Once they have determined the area of the roof that needs to be covered, students can perform simple multiplication and division problems to determine the number of bundles of shingles needed to cover the roof.

Solution:

Using the Pythagorean Theorem, students should first determine the length of the roof on one side of the ridge. $14^2 + 7^2 = 245$ and $\sqrt{245} \approx 15.65$.



Now, we can use the information given in the problem. We know the garage is 24 ft by 24 ft. Therefore, the length of one side of the garage is 24 ft. We know that there is a 2 ft overhang on all sides. The actual length of the garage is $24' + 4' = 28'$. In order to find the area of $\frac{1}{2}$ of the roof, multiply the length of the roof by the width of the roof (15.65, to get the area of $\frac{1}{2}$ of the roof). $28 \cdot 15.65 = 438.2$ sq. ft. The area of the entire roof is therefore $438.2 \cdot 2 = 876.4$ sq. ft.

We know from the problem that 3 bundles of shingles cover 100 sq. ft. of roof.

Therefore, we can determine that 1 bundle of shingles covers $100 \div 3 = 33.3$ sq. ft.

The number of single bundles required = area of roof $\div 33.3$.

$$876.4 \div 33.3 = 26.29.$$

The solution can also be determined using a similar manner. We know that 3 bundles cover 100 sq. ft. of roof so we can perform the following division problem:

$$876.4 \div 100 = 8.764$$

Since we know that it takes 3 bundles to cover that area we can then perform the following multiplication problem to determine the number of bundles used to cover the whole roof: $8.764 \cdot 3 = 26.292$.

Therefore, Karen should order 27 bundles.

Measurement 9-12:

- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders

Indiana State Standards:

G.7.7 Find and use measures of sides, volumes of solids, and surface areas of solids. Relate these measures to each other using formulas.

The Problem:

*The volume of a cube is 8 times less than the volume of another cube. What is the relationship of their surface areas?**

Mathematical Content:

This task requires students to compare the surface areas of two cubes given the difference in their volume. Utilizing the given information, students should be able to develop an equation for the volume of one of the cubes in terms of the other. Reducing the equation yields an equation for the length of the side of one cube in terms of the length of the side of the other. Using their knowledge of surface area and the equation for side lengths they derive, students may use substitution to determine the relationship of the surface areas of the cubes.

Solution:

One surface area is 24 times the surface area of the other cube. Let x represent the length of a side of the first cube and y represent the length of the side of a second cube. The volume of the first cube, x^3 , is 8 times less than the volume of the second cube, y^3 . So

$x^3 = \frac{1}{8}y^3 \rightarrow \sqrt[3]{8x} = y$. The surface area of the first cube is $6x^2$, while the surface area of

the second cube is $6y^2 = 6(\sqrt[3]{8x})^2 = 6(4)x^2 = 24x^2$. So the surface area of the second cube is 24 times the surface area of the first cube.

* Source: McGivney-Burelle, J. & White, J. (2005). August Calendar. *Mathematics Teacher*, 99 (1), 40.

GEOMETRY PROBLEMS

GEOMETRY RUBRIC

CATEGORY	1	2	3	4
Sketches	Sketches are difficult to understand or are not used. Sketches do not show understanding of the geometric concepts involved.	Sketches are not labeled or minimally labeled and are somewhat difficult to understand. Sketches do not show understanding of the geometric concepts involved.	Sketches not completely labeled but are clear and easy to understand. Sketches adequately display some understanding of the geometric concepts involved.	Sketches are completely labeled and are clear. Sketches demonstrate the student's understanding of the geometric concepts involved.
Mathematical Concepts	Explanation shows very limited understanding of the geometric concepts involved OR is not written. Student does not meet geometry expectations or standards.	Explanation shows some understanding of the geometric concepts needed to solve the problem. Student may partially meet standards and expectations addressed in the problem.	Explanation shows substantial understanding of the geometric concepts used to solve the problem. Student demonstrates adequate fulfillment of expectations and meets standards.	Explanation shows complete understanding of the geometric concepts used to solve the problem. Student demonstrates fulfillment of expectations and meets standards.
Mathematical Errors	More than 75% of the steps and solutions have mathematical errors.	Most (75-84%) of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solutions have no mathematical errors.	90-100% of the steps and solutions have no errors.
Explanation	Explanation is difficult to understand and is missing several components OR was not included.	Explanation is a little difficult to understand, but includes critical components.	Explanation is clear.	Explanation is detailed and clear.
Mathematical Terminology and Notation	There is little use, or a lot of inappropriate use, of geometric terminology and notation.	Correct terminology and notation is used, but it is sometimes not easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminology and notation are always used, making it easy to understand what was done.
Neatness and Organization	The work appears sloppy and unorganized. It is hard to determine what information goes together.	The work is presented in an organized fashion but may be hard to read at times.	The work is presented in a neat and organized fashion that is usually easy to read.	The work is presented in a neat, clear, organized fashion that is always easy to read.

Geometry 9-12:

- Use Cartesian coordinates and other coordinate systems such as navigational, polar, or spherical systems, to analyze geometric situations

Indiana State Standards:

G.1.4 Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.

The Problem:

Let S be the set of points (a, b) in the coordinate plane, where each of a and b may be -1 , 0 , or 1 . How many distinct lines pass through at least two members of S ?^{*}

Mathematical Content:

This task requires students to visualize lines in the coordinate plane. Students must think about the lines possible given the situation described. There are three numbers that can be used for the a coordinate and three more for the b coordinate. Students may use counting techniques to determine the total number of lines created using two points or simply write down the set of points that determine individual lines. Of the lines created, students must determine how many of them are distinct.

Solution:

20. There are $\binom{9}{2} = 36$ pairs of points in S , and each pair determines a line. However, three horizontal, three vertical, and two diagonal lines pass through three points of S , and these lines are each determined by three different pairs of points in S . Thus the number of distinct lines is $36 - 2 \cdot 8 = 20$.

^{*} Source: McGivney-Burelle, J. & White, J. (2005). September Calendar. *Mathematics Teacher*, 99 (2), 112.

Geometry 9-12:

- Analyze properties and determine attributes of two- and three-dimensional objects
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations
- Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates

Indiana State Standards:

- G.1.1 Find the lengths and midpoints of line segments in one- or two-dimensional coordinate systems.
- G.4.2 Define, identify, and construct altitudes, medians, angle bisectors, and perpendicular bisectors.
- G.6.1 Find the center of a given circle. Construct the circle that passes through three given points not on a line.
- G.6.4 Construct tangents to circles and circumscribe and inscribe circles.

The Problem:

*A triangle with sides of 5, 12, and 13 has both an inscribed and a circumscribed circle. What is the distance between the centers of those circles?**

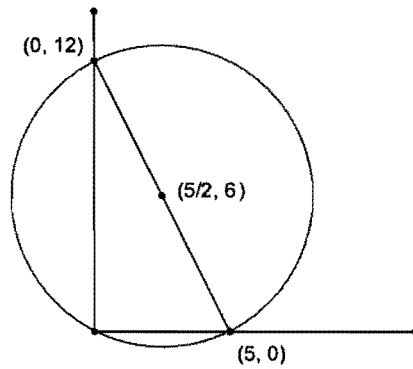
Mathematical Content:

This task requires students to work with geometric shapes in the coordinate plane. Students must construct the circumscribed and inscribed circles of a triangle given its side lengths. Students must then find the centers of the two circles. After finding the centers students can use the distance formula to find the distance between them.

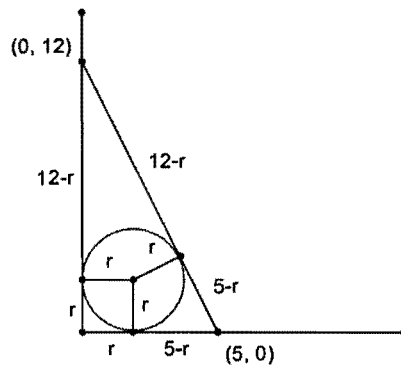
Solution:

$\frac{\sqrt{65}}{2}$. The triangle is a right triangle that can be placed in a coordinate system with vertices at (0, 0), (5, 0), and (0, 12), as shown in the figure. The center of the circumscribed circle is the midpoint of the hypotenuse, which is $\left(\frac{5}{2}, 6\right)$.

* Source: McGivney-Burelle, J. & White, J. (2005). September Calendar. *Mathematics Teacher*, 99 (2), 112.



To determine the radius r of the inscribed circle notice that by symmetry, there is a pair of congruent triangles whose legs are of length $12 - r$ and r , and another pair of congruent right triangles with legs of length $5 - r$ and r . See the second figure. Upon reflection, the length of the hypotenuse can be shown to be the sum of the longer legs: $(12 - r) + (5 - r)$. From Pythagorean theorem we know that the hypotenuse has a length of 13 units, so $(12 - r) + (5 - r) = 13 \rightarrow r = 2$.



Therefore, the center of the inscribed circle is $(2, 2)$, and the distance between the two centers is $\sqrt{\left(\frac{5}{2} - 2\right)^2 + (6 - 2)^2} = \frac{\sqrt{65}}{2}$.

Geometry 9-12:

- Analyze properties and determine attributes of two- and three-dimensional objects
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations
- Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates
- Draw and construct representations of two- and three-dimensional objects using a variety of tools

Indiana State Standards:

- G.1.2 Construct congruent segments and angles, angle bisectors, and parallel and perpendicular lines using a straight edge and compass, explaining and justifying the process used.
- G.1.4 Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.
- G.4.1 Identify and describe triangles that are right, acute, obtuse, scalene, isosceles, equilateral, and equiangular.
- G.5.6 Solve word problems involving right triangles.

The Problem:

*Two points, $A(2, 0)$ and $B(3, 5)$, are given. Find the third point $C(0, c)$ such that $\triangle ABC$ is a right triangle. Is this triangle isosceles?**

Mathematical Content:

This task requires students to recall their knowledge of the properties of triangles. Students must first find a point C such that $\triangle ABC$ a right triangle. Once students have done so, they can use the properties of perpendicular lines to determine whether or not their point leads to $\triangle ABC$ being isosceles.

Solution:

$C_1(0, 2)$, $C_2(0, 3)$, $C_3\left(0, \frac{2}{5}\right)$, $C_4\left(0, \frac{28}{5}\right)$ are all possible locations for the third point.

$\triangle ABC_2$ is a right isosceles triangle ($AC_2 = BC_2 = \sqrt{13}$).

Using the property of perpendicular lines, $m_1 m_2 = -1$, where m_1 and m_2 are slopes

$m = \frac{(y_1 - y_2)}{(x_1 - x_2)}$, we begin by examining $m_{AB} = 5$, $m_{BC} = \frac{5-c}{3}$, $m_{AC} = \frac{-c}{2}$. If AB is the

hypotenuse from $m_{AC} m_{BC} = -1$, then $\frac{5-c}{3} \cdot \frac{-c}{2} = -1$. By solving this equation we can

find two points: $C_1(0, 2)$, $C_2(0, 3)$. If AB is the length of a side then $m_{AC} = \frac{-c}{2} = \frac{-1}{5}$ or

* Source: McGivney-Burelle, J. & White, J. (2005). September Calendar. *Mathematics Teacher*, 99 (2), 112.

$m_{BC} = \frac{5-c}{3} = \frac{-1}{5}$. Solving each of these equations leads to two other points:

$$C_3\left(0, \frac{2}{5}\right), C_4\left(0, \frac{28}{5}\right).$$

Geometry 9-12:

- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems to analyze geometric situations
- Understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices
- Use various representations to help understand the effects of simple transformations and their compositions

Indiana State Standards:

G.2.4 Apply transformations (slides, flips, turns, expansions, and contractions) to polygons to determine congruence, similarity, symmetry, and tessellations. Know that images formed by slides, flips, and turns are congruent to the original shape.

The Problem:

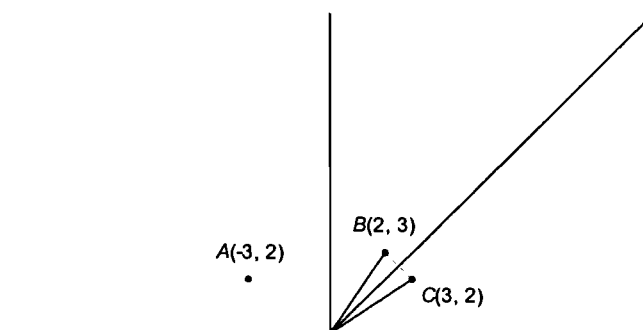
*The point $A(-3, 2)$ is rotated by 90° clock-wise around the origin to point B . Point B is then reflected over the line $y = x$ to point C . What are the coordinates of B and C ?**

Mathematical Content:

This task requires students to have an understanding of various transformations and their effects on a point. Students must perform a series of transformations on the given point. Once the transformations have been performed students must determine the new coordinates of the point.

Solution:

$(3, 2)$. The rotation takes $A(-3, 2)$ to $B(2, 3)$, and the reflection takes B to $C(3, 2)$, as shown.



* Source: McGivney-Burelle, J. & White, J. (2005). August Calendar. *Mathematics Teacher*, 99 (1), 40.

DATA ANALYSIS AND PROBABILITY PROBLEMS

DATA ANALYSIS AND PROBABILITY RUBRIC

CATEGORY	1	2	3	4
Mathematical Concepts	Explanation shows very limited understanding of the concepts involved OR is not written. Student does not meet expectations or standards.	Explanation shows some understanding of the concepts needed to solve the problem. Student may partially meet standards and expectations addressed in the problem.	Explanation shows substantial understanding of the concepts used to solve the problem. Student demonstrates adequate fulfillment of expectations and meets standards.	Explanation shows complete understanding of the concepts used to solve the problem. Student demonstrates fulfillment of expectations and meets standards.
Mathematical Errors	More than 75% of the steps and solutions have mathematical errors.	Most (75-84%) of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solutions have no mathematical errors.	90-100% of the steps and solutions have no errors.
Explanation	Explanation is difficult to understand and is missing several components OR was not included.	Explanation is a little difficult to understand, but includes critical components.	Explanation is clear.	Explanation is detailed and clear.
Mathematical Terminology and Notation	There is little use, or a lot of inappropriate use, of correct mathematical terminology and notation.	Correct terminology and notation is used, but it is sometimes not easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminology and notation are always used, making it easy to understand what was done.
Neatness and Organization	The work appears sloppy and unorganized. It is hard to determine what information goes together.	The work is presented in an organized fashion but may be hard to read at times.	The work is presented in a neat and organized fashion that is usually easy to read.	The work is presented in a neat, clear, organized fashion that is always easy to read.

Data Analysis and Probability 6-8:

- Use proportionality and basic understanding of probability to make and test conjectures about the results of experiments and simulations

Indiana State Standards:

- 6.6.6 Understand and represent probabilities as ratios, measures of relative frequency, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable.
- 7.6.4 Analyze data displays, including ways that they can be misleading. Analyze ways in which the wording of questions can influence survey results.
- 8.6.1 Identify claims based on statistical data and, in simple cases, evaluate the reasonableness of the claims. Design a study to investigate the claim.

The Problem:

Does drinking soda affect your health?

*For this study, researchers questioned ninth and tenth-grade girls at a Boston-area high school. Do the data support the headline?**

HEADLINE: "Teenage girls' soda intake is linked to broken bones."		
	<i>Fractures</i>	<i>No Fractures</i>
<i>Drank Cola</i>	38	69
<i>Did Not Drink Cola</i>	5	52

Hint: What percentage of the teenage girls who drank cola also had fractures?

Mathematical Content:

This task asks students to interpret the data from the table to determine whether or not the data actually supports the given headline. In order to do so, students must develop a method for determining whether or not one factor seems to have an effect on the other.

Solution:

One way to analyze this situation is to consider two groups of teens in the study: those who drank soda, and those who did not. From the table, there are $38 + 69 = 107$ students who drank cola, and $5 + 52 = 57$ who did not. Of the 107 students who drank cola, 38 had fractures, or approximately 36%. Of the 57 students who did not drink cola, 5 had fractures, or about 9%. The information is summarized in the table on the next page.

* Source: National Council of Teachers of Mathematics. (2004). Math Challenge #68, Bones. *Figure This!* Retrieved August 8, 2005 from <http://www.figurethis.org/challenges/c68/challenge.htm>

	Fractures	No Fractures
Drank Cola	$\frac{38}{38 + 69} \approx 36\%$	$\frac{69}{38 + 69} \approx 64\%$
Did Not Drink Cola	$\frac{5}{5 + 52} \approx 9\%$	$\frac{52}{5 + 52} \approx 91\%$

Based on this information, students who drink cola seem to be more likely to have fractures. Just because an association exists, however, does not mean that drinking cola causes fractures. Other factors may be involved.

Data Analysis and Probability 6-8:

- Use proportionality and basic understanding of probability to make and test conjectures about the results of experiments and simulations

Indiana State Standards:

- 6.6.6 Understand and represent probabilities as ratios, measures of relative frequency, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable.
- 7.6.2 Make predictions from statistical data.

The Problem:

Who's on first today?

In May 1999, two National League baseball players, Joe McEwing of the St. Louis Cardinals and Mike Lieberthal of the Philadelphia Phillies, each had the batting averages shown below.

<i>Player</i>	<i>Team</i>	<i>At Bats</i>	<i>Hits</i>	<i>Batting Average</i>
<i>M. Lieberthal</i>	<i>Phillies</i>	<i>132</i>	<i>45</i>	<i>.341</i>
<i>J. McEwing</i>	<i>Cardinals</i>	<i>132</i>	<i>45</i>	<i>.341</i>

*Suppose McEwing then batted .800 (4 hits in 5 at bats), and Lieberthal was perfect (3 hits in 3 at bats). Which player now has the higher batting average? Are you surprised?**

Hint: Batting average = $\frac{\text{Number of hits}}{\text{Number of at bats}}$ (rounded to the nearest thousandth)

Mathematical Content:

This task requires students to organize information and develop a method for determining which batter has the higher batting average after their additional hits. Students should recognize that this problem involves proportional reasoning and use proportions accordingly.

Solution:

Both players had 45 hits in 132 at bats. Then with the statistics from the next at bats, McEwing's average is $\frac{49}{137} \approx .358$ while Lieberthal's batting average is $\frac{48}{135} \approx .356$.

* Source: National Council of Teachers of Mathematics. (2004). Math Challenge #14, Battling Averages. *Figure This!* Retrieved August 8, 2005 from <http://www.figurethis.org/challenges/c14/challenge.htm>

Player	Team	At Bats	Hits	Batting Average	Next At Bats	Next Hits	New Average
M. Lieberthal	Phillies	132	45	$\frac{45}{132} \approx .341$	3	3	$\frac{48}{135} \approx .356$
J. McEwing	Cardinals	132	45	$\frac{45}{132} \approx .341$	5	4	$\frac{49}{137} \approx .358$

McEwing has the higher batting average. One way to make sense of this unexpected result is to imagine that McEwing gets 3 hits in his first 3 at bats while Lieberthal also gets 3 hits in 3 at bats. Then the pair is still tied. During McEwing's last 2 at bats, he gets 1 hit. This average of 1 for 2, or .500, is better than his current average, so his batting average goes up.

Data Analysis and Probability 6-8:

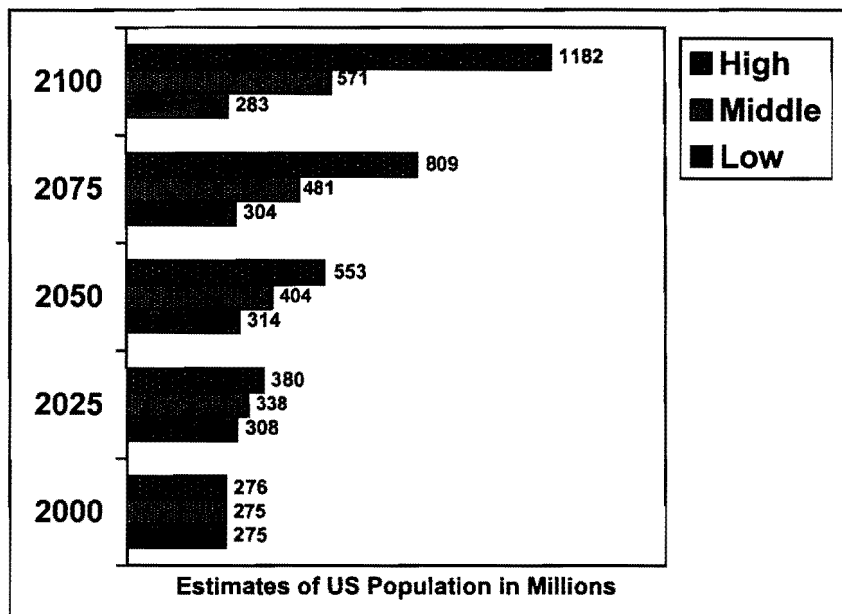
- Select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots
- Discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatterplots

Indiana State Standards:

8.6.1 Identify claims based on statistical data and, in simple cases, evaluate the reasonableness of the claims. Design a study to investigate the claim.

The Problem:

How many people live in the United States?



*The US Census Bureau estimates that the US population may double in the next 100 years. Other experts have suggested that the population may increase by 400% over the same period. Does the graph support these statements?**

Mathematical Content:

This task requires students to investigate various claims. Students must recognize what it means for something to “double” in value and what it to increase by “400%.” Students

* Source: National Council of Teachers of Mathematics. (2004). Math Challenge #53, Census. *Figure This!* Retrieved August 8, 2005 from <http://www.figurethis.org/challenges/c53/challenge.htm>

will use this information to determine whether or not any of the claims are close to the experts' predictions.

Solution:

The estimated US population in 2000 was about 275 million people. Note that a 100% increase is the same as doubling which gives 550 million people. An increase of 400% would mean the 2000 population plus 4 times the population.

$275,000,000 + (4 \cdot 275,000,000)$ or $5 \cdot 275,000,000$ which is 1,375,000,000 people.

Comparing the doubled 200 population and projected 2100 population, the low estimate is the only one that is not at least doubled. None of the estimates in the graph supports a 400% increase.

	Low Estimate	Middle Estimate	High Estimate
2000	275,000,000	275,000,000	275,000,000
2100	283,000,000	572,000,000	1,182,000,000
Double the 2000 population	550,000,000	550,000,000	552,000,000
400% increase of the 2000 population	1,375,000,000	1,375,000,000	1,380,000,000

Data Analysis and Probability 6-8:

- Select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots
- Find, use, and interpret measures of center and spread, including mean and interquartile range

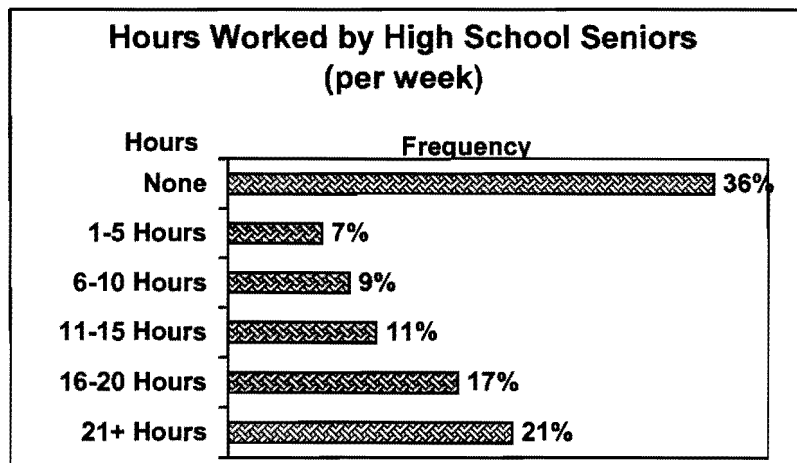
Indiana State Standards:

- 6.6.2 Make frequency tables for numerical data, grouping the data in different ways to investigate how different groupings describe the data. Understand and find relative and cumulative frequency for a data set. Use histograms of the data and of the relative frequency distribution, and a broken line graph for cumulative frequency, to interpret the data.
- 6.6.3 Compare the mean, median, and mode for a set of data and explain which measure is most appropriate in a given context.
- 7.6.1 Analyze, interpret, and display data in appropriate bar, line, and circle graphs and stem-and-leaf plots and justify the choice of display.
- 8.6.3 Understand the meaning of, and be able to identify or compute the minimum value, the lower quartile, the median, the upper quartile, the interquartile range, and the maximum value of a data set.

The Problem:

How much time do teens spend on the job?

*What's the average number of hours high school seniors work per week?**



Hint: Assume that there are 100 students in the survey data.

* Source: National Council of Teachers of Mathematics. (2004). Math Challenge #48, Working Hours. *Figure This!* Retrieved August 8, 2005 from <http://www.figurethis.org/challenges/c48/challenge.htm>

Mathematical Content:

This task requires students to determine the average number of hours high school seniors work per week from a display of data. Students must recognize what the display represents and use it accordingly.

Solution:

There are many ways to do this problem. The answer can only be approximated because of the way the data are reported.

If you assume that the table shows data for 100 students, then the 7% who worked from one to five hours would correspond to seven students who worked one to five hours. To find an average (mean), you need to estimate the total number of hours worked. Consider the seven students who worked from one to five hours. Taken together, the least time they could have worked is seven hours. The greatest they could have worked is 35 hours. (the actual number is probably somewhere in between these values.) The first chart shows the least number of hours the 100 students could have worked, while the second chart shows the greatest number of hours they could have worked.

Least Number of Hours	Number of Students*	Least Number of Total Hours
0	36	0
1	7	7
6	9	54
11	11	121
16	17	272
21	21	441
Total	100	895

*Estimated

Greatest Number of Hours	Number of Students*	Greatest Number of Total Hours
0	36	0
5	7	35
10	9	90
15	11	165
20	17	340
25*	21	525
Total	100	1155

*Estimated

To find the lower value for the average number of hours worked per week, divide the total number of hours by the number of students: $\frac{895}{100} = 8.95$. The larger value for the

average work week can be found in the same way: $\frac{1155}{100} = 11.55$. Using this approach,

an estimate for the average for a high school senior is between 8.95 hours and 11.55 hours.

Another way to estimate the average uses an average for the hours worked in each category.

Smallest Number of Hours	Largest Number of Hours	Average Number of Hours	Number of Students*	Total Hours Worked
0	0	0	36	0
1	5	3	7	21
6	10	8	9	72
11	15	13	11	143
16	20	18	17	306
21	25*	23	21	483
		Total	100	1025

For find a value for the average, divide the total number of hours by the number of students $\frac{1025}{100} = 10.25$ This method results in an average of about 10.25 hours.

Data Analysis and Probability 6-8, 9-12:

- Compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models
- Understand the concepts of conditional probabilities and independent events
- Understand how to compute the probabilities of a compound event

Indiana State Standards:

- 6.6.4 Show all possible outcomes for compound events in an organized way and find the theoretical probability of each outcome.
- 7.6.7 Find the number of possible arrangements of several objects using a tree diagram.
- 8.6.7 Find the number of possible arrangements of several objects by using the Basic Counting Principle.
- PS.2.1 Understand the counting principle, permutations, and combinations and use them to solve problems.

The Problem:

I forgot the combination! I'm ready to cry. How many combinations will I have to try?

A combination lock uses three numbers from 0 to 39. It opens when these numbers are dialed in a particular order: right, left, right. How many possible combinations are there?

Hint: Think about how many choices you have before dialing each number.

Additional Related Problems:

Suppose the combination for a particular brand of lock allows each number to appear only once. If the lock uses three numbers from 0 to 39 (as in the challenge), how many combinations are possible?

*Suppose the combination for a bicycle lock is 10-24-32. With this lock, however, the numbers on either side work as well as the actual number. For example, the combinations 9-25-33 and 11-23-22 will also open the lock. How many different combinations will open this particular lock?**

Mathematical Content:

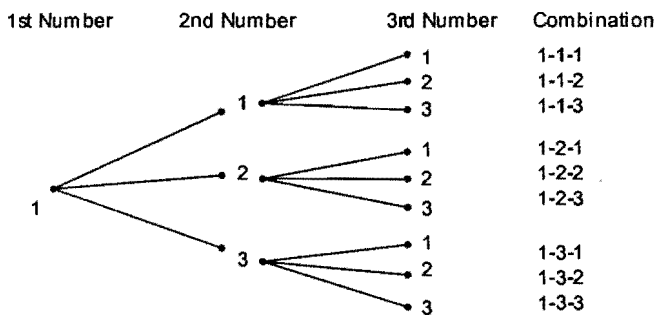
This task requires students to investigate compound events. The first question requires students to determine the number of possible combinations with replacement, the second question asks students to find the number of possible combinations without replacement,

* Source: National Council of Teachers of Mathematics. (2004). Math Challenge #22, Combination Locks. *Figure This!* Retrieved August 8, 2005 from <http://www.figurethis.org/challenges/c22/challenge.htm>

and the last question asks students to determine how many combinations will open the lock described.

Solution:

In the challenge, the lock uses the numbers 0 to 39. Start with an easier problem using 1, 2, and 3. You can count the possibilities by drawing a tree diagram. A portion of this tree diagram is shown below



If you start with 1, you get nine different combinations. If you start with 2 instead of 1, you also get 9 combinations. If you start with 3, you get 9 more possibilities for a total of $9 + 9 + 9 = 27$ different combinations. Think about this in terms of choices. You can choose any of three numbers as a possible first number in the combination, follow that with a choice of any of the three numbers as the second number in the combination, and finally choose any of the three numbers for the final number in the combination for a total of $3 \cdot 3 \cdot 3 = 27$ different combinations. If the lock used the numbers 1, 2, 3, and 4, you would have to choose from four numbers, three different times. This would give you $4 \cdot 4 \cdot 4 = 64$ choices for the combination. The lock in the challenge requires that you choose from 40 different numbers, three different times. Therefore, there are $40 \cdot 40 \cdot 40 = 64000$ different combinations.

The next question asks you to find the number of combinations if each number can only be selected once. Therefore, each time you choose a number the amount of numbers you can choose decreases by one. $40 \cdot 39 \cdot 38 = 59280$ different combinations.

The last question asks you to find the number of combinations that would work given the fact that the numbers on either side of the actual combination work. In this case, there are three choices for each of the three numbers in the combination. So the answer is $3 \cdot 3 \cdot 3 = 27$.

Data Analysis and Probability 6-8, 9-12:

- Compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models
- Understand the concepts of conditional probabilities and independent events
- Understand how to compute the probabilities of a compound event

Indiana State Standards:

- 6.6.4 Show all possible outcomes for compound events in an organized way and find the theoretical probability of each outcome.
- 7.6.7 Find the number of possible arrangements of several objects using a tree diagram.
- 8.6.7 Find the number of possible arrangements of several objects by using the Basic Counting Principle.
- PS.2.1 Understand the counting principle, permutations, and combinations and use them to solve problems.

The Problem:

*Henry's Hamburger Heaven offers its hamburgers with ketchup, mustard, mayonnaise, tomato, lettuce, pickles, cheese, and onions. A customer can choose one, two, or three meat patties and any assortment of condiments. How many different kinds of hamburgers can be ordered?**

Mathematical Content:

This task requires students to determine the number of different kinds of hamburgers that can be made given the details about the options available. Students can break this problem down into two parts: first they must find the number of combinations of condiments and then number of options available for hamburger patties. After finding the number of combinations of condiments, this problem is reduced to a simple multiplication problem. That is, students must multiply the number of combinations of condiments by the number of choices for meat patties.

Solution:

768. A customer makes one of two choices for each condiment, to include it or not to include it. The choices are made independently, so there are $2^8 = 256$ possible combinations of condiments. For each of those combinations there are three choices regarding the number of meat patties, or $(3)(256) = 768$ different kinds of hamburgers.

* Source: McGivney-Burelle, J. & White, J. (2005). August Calendar. *Mathematics Teacher*, 99 (1), 40.

Data Analysis and Probability 9-12:

- Understand the concepts of sample space and probability distribution and construct sample spaces and distributions in simple cases
- Understand the concepts of conditional probabilities and independent events
- Understand how to compute the probabilities of a compound event

Indiana State Standards:

PS.2.1 Understand the counting principle, permutations, and combinations and use them to solve problems.

PS.2.2 Understand and use the addition rule to calculate probabilities for mutually exclusive and nonmutually exclusive events.

The Problem:

*Two 8-sided dice each have faces numbered 1 through 8. When the dice are rolled, each face has an equal probability of appearing on the top. What is the probability that the product of the two numbers on the top of the dice is greater than their sum?**

Mathematical Content:

This task requires students to determine the probability that the product of two numbers rolled on two dice is greater than their sum. Students should determine the number of pairs that can be discarded because they do not meet the requirements described. Upon doing so, the problem is reduced to a simple subtraction problem.

Solution:

The probability is $\frac{3}{4}$. There are $8 \cdot 8 = 64$ ordered pairs that can represent the top numbers on the two dice. Let m and n represent the top numbers on the dice. Then $mn > m + n$ implies that $mn - m - n > 0$, that is, $mn - m - n + 1 > 1$. Since $mn - m - n + 1 = (m - 1) \cdot (n - 1)$, then $(m - 1) \cdot (n - 1) > 1$. Since $m > 0$ and $n > 0$, this inequality is satisfied except when $m = 1$, $n = 1$, or $m = n = 2$. Sixteen total ordered pairs (m, n) are excluded by these conditions (when $m = 1$: 8 pairs; $n = 1$: 7 additional pairs; and $m = n = 2$: 1 more pair), so the probability that the product is greater than the

sum is $\frac{64 - 16}{64} = \frac{48}{64} = \frac{3}{4}$.

* Source: McGivney-Burelle, J. & White, J. (2005). August Calendar. *Mathematics Teacher*, 99 (1), 40.

Data Analysis and Probability 9-12:

- Understand the concepts of sample space and probability distribution and construct sample spaces and distributions in simple cases
- Understand the concepts of conditional probability and independent events

Indiana State Standards:

PS.2.1 Understand the counting principle, permutations, and combinations and use them to solve problems.

PS.2.3 Understand and use the multiplication rule to calculate probabilities for independent and dependent events.

The Problem:

*An integer x , with $10 \leq x \leq 99$, is to be chosen. If all choices are equally likely, what is the probability that at least one digit of x is a 7?**

Mathematical Content:

This task requires students to investigate the digits between 10 and 99 for numbers that have at least one 7 in them. This task can easily be broken down into steps and solved rather quickly.

Solution:

1/5. There are 90 possible choices for x . Ten of these have a units digit of 7 and nine have a tens digit of 7. Because 77 has been counted twice, there are $10 + 9 - 1 = 18$ choices for x for which at least one digit is a 7. Therefore the probability is $18/90 = 1/5$.

* Source: McGivney-Burelle, J. & White, J. (2005). September Calendar. *Mathematics Teacher*, 99 (2), 112.

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